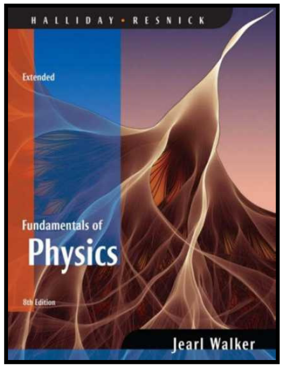
Workshop Physics

1017 - 311

University Physics I



Week 3 : Day 3

Projectile Motion Equations

- I most situations the 2D Kinematics equations will reduce to the following:
  - Position

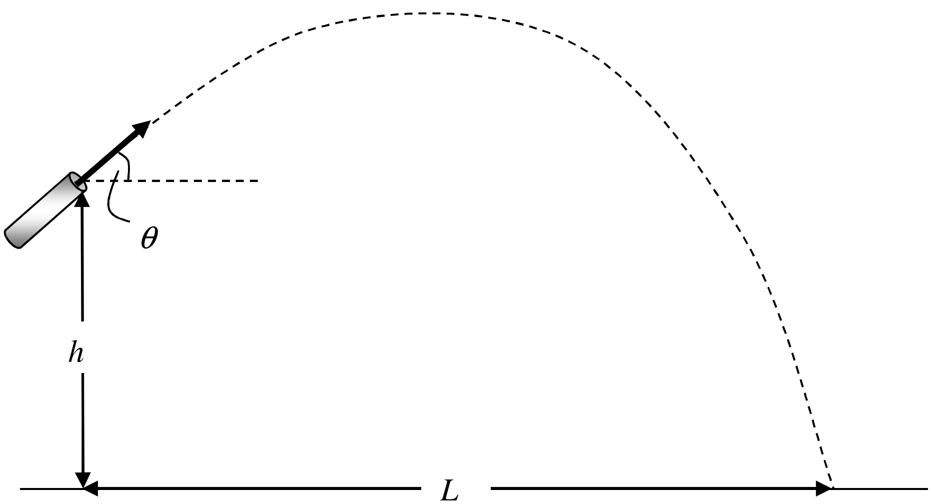
$$x = x_0 + v_{0x}t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

  - Velocity

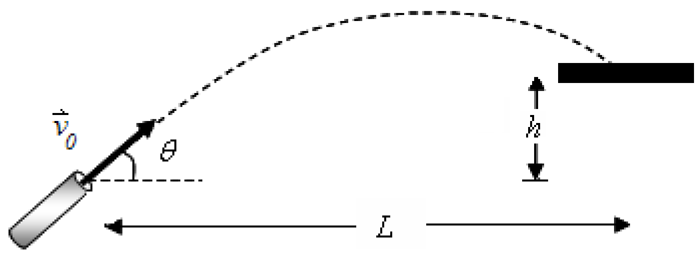
$$v_x = v_{0x}$$

$$v_y = v_{0y}t - gt$$



Example – Hitting a Platform

- **Cannon shot with following parameters**
  - Initial velocity of 15.0 m/s
  - 60° above the horizontal
  - Lands on a platform 8.0 m above
  - Find the horizontal distance



$$v_y^2 = v_{0y}^2 + 2a(y - y_0)$$

$$\Rightarrow v_y = -\sqrt{v_0^2 \sin^2(\theta) - 2gh}$$

$$\Rightarrow v_y = -3.46\, m/s$$

$$v_y = v_{0y} - gt$$

$$\Rightarrow v_y = v_0 \sin(\theta) - gt$$

$$\Rightarrow t = \frac{v_0 \sin(\theta) - v_y}{g} = \frac{12.99 + 3.46}{9.8} = 1.68\, s$$

$$x = x_0 + v_{0x} t \quad \Rightarrow L = v_0 \cos(\theta) t = 12.6\, m$$

More Projectile Problems…

- **Practice the Set up**
  - You never know what you might see on a quiz…

If $a_x = 0$, v_x = constant, $x = x_0 + v_x t$

If a_y = constant, $v_y = v_{y0} + a_y t$ $\quad y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$ $\quad v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ $\quad y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$

For projectile problems, a_y = -9.8 m/s^2

Projectile Problems

1. A rock is thrown horizontally with initial speed of 4.50 m/s from a table that is 1.50 m tall. At what horizontal distance does it hit the floor?

2. A rock is thrown horizontally from a table that is 1.50 m tall and hits the floor at a horizontal distance of 3.00 m. What is the initial velocity of the rock?

3. A diver springs upward from the end of a board that is 5.0 m above the water, with an initial velocity of 4.0 m/s at an angle of 60.0º above the horizontal.

   Calculate the x and y components of the velocity with which the diver enters the water.

   At what angle does the diver enter the water?

4. A diver springs upward from the end of a board that is some distance above the water, with an initial velocity of 4.0 m/s at an angle of 60.0º above the horizontal. She enters the water at a horizontal distance of 3.5 m.

   What is the height of the diving board?

5. A plane traveling at 55.0 m/s is diving at an angle of 25.0° below the horizontal when it releases an aid package. The package hits at a point 245.0 m horizontally from its release point. From what height was it dropped?

6. A ball is thrown upwards at an angle of 30.0° from the roof of a building 20.0 m tall and lands a distance 30.0 m from the base of the building. Assume that it is in free-fall throughout, and determine
   (a) the initial speed of the ball
   (b) the maximum height reached by the ball.

7. A basketball is released from a height of 2.00 m above the floor and enters a basket that is 3.0 m above the floor. The ball is shot at an angle of 53.0° with a speed of 8.00 m/s. How far was the shooter from the basket?

Projectile Predictions

❑ Predicting Projectile Motion

- Find Muzzle velocity
  - *Horizontal cannon*
- Predict the Range
  - *Tilted cannon*

Predicting the motion of a projectile

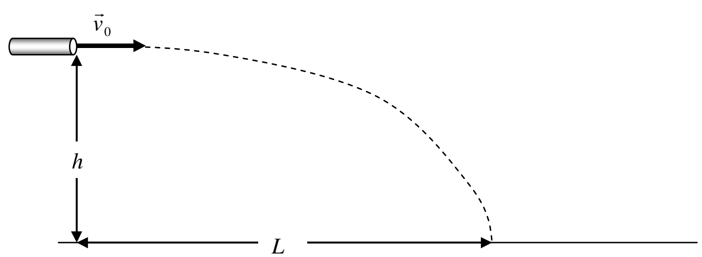
These problems deal with the theory behind the experiments we will try in the next workshop. Keep this worksheet and any other sheets used, you will need them later.

Case 1

Let's begin with the special case in which a ball is fired horizontally from a gun. The gun is at a height h above the ground and the ball leaves the barrel with a velocity $\vec{v}_0$, as shown in the diagram. Where will the ball land?

Your job is to derive an expression for the horizontal distance (L) from the gun to the landing point in terms of the height h and the magnitude of the initial velocity $|\vec{v}_0|$ (which you can just call v_0). Find another expression for the time of flight, t.

Remember that, during its motion, the ball is subject to a downwards acceleration, g.



Activity – Projectile Experiment

❑ Projectile Experiment

- ➢ Find Muzzle velocity
  - *Calculate uncertainty*
- ➢ Predict the Range
  - *Calculate uncertainty*
- ➢ Group Reports
  - *Due at end of class*

Projectile Experiment

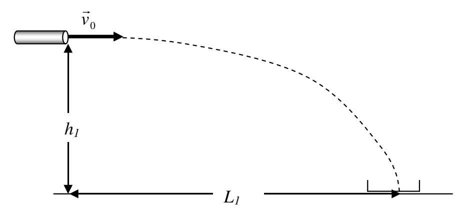
Earlier you derived an equation showing how to find the initial velocity of a projectile fired horizontally by measuring the initial height, h, and the range (L). You also developed an expression for the range of a projectile fired with initial velocity v_0 from a height h at an angle θ. You will now do the experiment using a spring-loaded gun, and will determine muzzle velocity and predicted range along with uncertainties in these values. Finally, you will test the results of your prediction.

Each table will have one such spring-loaded gun.

DETERMINING MUZZLE VELOCITY

Clamp the gun to your table. Gently push the ball to the **medium range** setting using the rod provided. Fire a shot or two to get a rough idea of the range and then set up a catch-box that will catch the ball and record the position where it hits. In the catch-box place a sheet of carbon paper face up, covered by a piece of regular paper, and taped in place. Be sure you can reset the box to its original position, since it may move when hit by the ball.

First measure the height of the cannon above the floor (h_1) and its uncertainty. Think carefully about what vertical height is to be measured. Should you measure from the floor to the bottom of the ball, center of the ball, or top of the ball?



DISCUSS WITH AN INSTRUCTOR BEFORE PROCEEDING.

Taking care that the ballistic pendulum device does not move, fire 3 shots into the cardboard box: the carbon paper will indicate the landing point. Measure as accurately as possible the horizontal distance traveled by the projectile (L_1), and calculate its uncertainty, ΔL_1.

Use your measurements of h_1 and L_1 and their uncertainties to determine

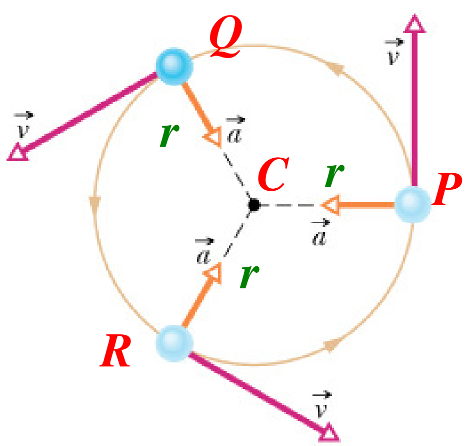
1. the muzzle velocity (v_0) of the projectile,
2. the absolute uncertainty in v_0.

The expression for the muzzle velocity will involve multiplication/division, and so relative (fractional) errors are what we are concerned with. Determine the fractional errors in height and range and decide whether you need to include both in your uncertainty propagation. At the end you need the absolute uncertainty in muzzle velocity.

SHOW YOUR CALCULATION TO AN INSTRUCTOR BEFORE PROCEEDING.

Uniform Circular Motion

A particle is in uniform circular motion if it moves on a circular path of radius r with constant **speed** v. Even though the speed is constant, the velocity is not. The reason is that the direction of the velocity vector changes from point to point along the path. The fact that the velocity changes means that the acceleration is not zero. The acceleration in uniform circular motion has the following characteristics:



1. Its vector points toward the center of the circle
2. Its magnitude a is given by the equation

$$a = \frac{v^2}{r}.$$

The time T it takes to complete a full revolution is known as the "period." It is given by the equation:

$$T = \frac{2\pi r}{v}.$$

Centripetal Acceleration I

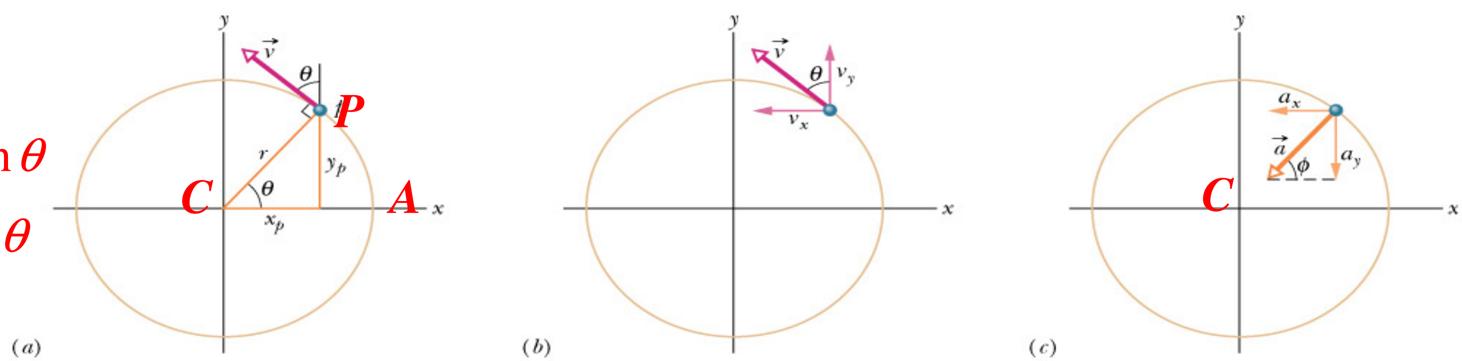
- To demonstrate these relationships consider the following situation
  - The velocity components at the point P are:

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v\sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}} \qquad \sin\theta = \frac{y_P}{r} \qquad \cos\theta = \frac{x_P}{r}$$

Here x_P and y_P are the coordinates of the rotating particle.

$$v_x = -v\sin\theta$$

$$v_y = v\cos\theta$$

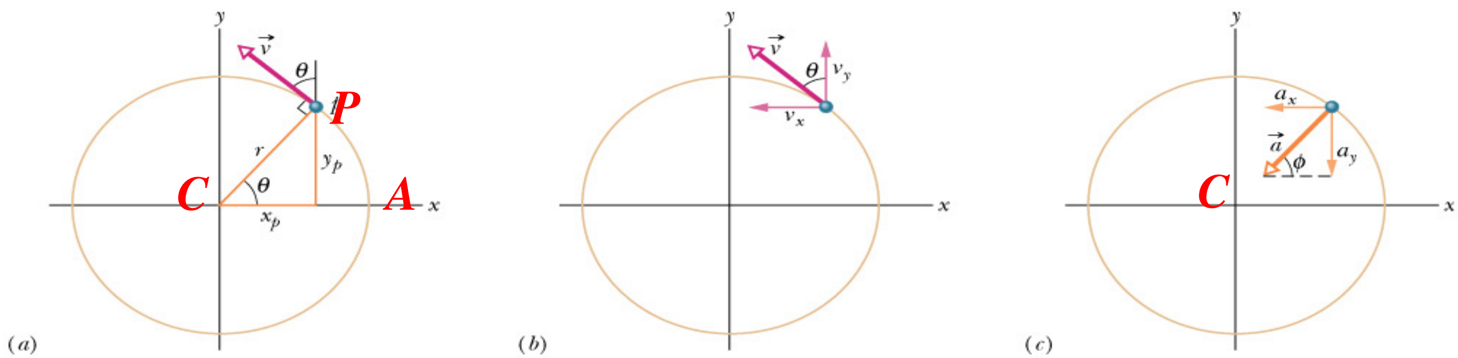


$$\vec{v} = \left(-v\frac{y_P}{r}\right)\hat{\mathrm{i}} + \left(v\frac{x_P}{r}\right)\hat{\mathrm{j}}. \quad \text{Acceleration } \vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_P}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_P}{dt}\right)\hat{\mathrm{j}}.$$

Centripetal Acceleration II

We note that $\dfrac{dy_P}{dt} = v_y = v\cos\theta$ and $\dfrac{dx_P}{dt} = v_x = -v\sin\theta$.

$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}} \qquad a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}$$



$$\tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta \rightarrow \phi = \theta \rightarrow \vec{a} \text{ points toward } C.$$

Activity – Circular Motion Problems

- **An automobile whose speed is increasing at a rate of 0.600 m/s2 travels clockwise along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find**
  - the tangential acceleration component
  - the radial acceleration component, and
  - the magnitude and direction of the total acceleration
  - Sketch the acceleration components.

Circular Motion

An automobile whose speed is increasing at a rate of 0.600 m/s^2 travels clockwise along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find

a) the tangential acceleration component
b) the radial acceleration component, and
c) the magnitude and direction of the total acceleration

Sketch the acceleration components.