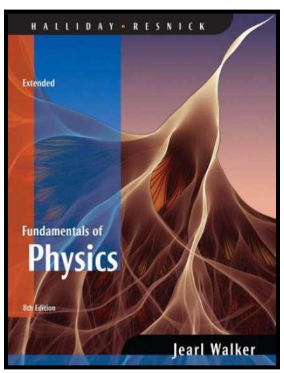
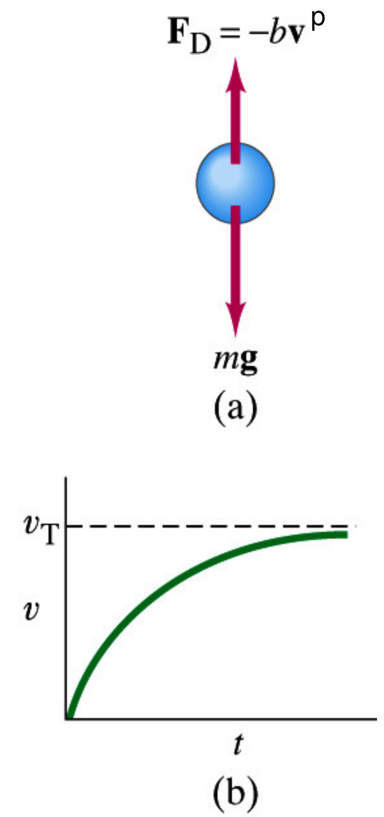
University Physics I

Week 5 : Day 3



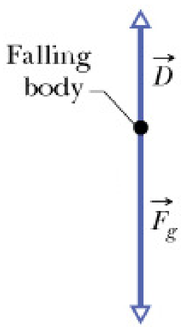
Air "Friction" or Drag

- Objects that move through the air also experience a "friction" type force.
- The drag force has the following properties:
  - It is proportional to the cross sectional area of the object.
  - It is proportional to the velocity of the object.
  - It is directed in a direction opposite to the direction of motion.
- The drag force is responsible for the object reaching a terminal velocity (when the drag force balances the gravitational force).



Equations of Motion

- **For a body falling subject to a general drag force we can use Newton's Third law to derive the velocity profile of the object.**
  - Begin with the definition



$$\sum F_y = -b(-v^p) - mg = m(-a)$$

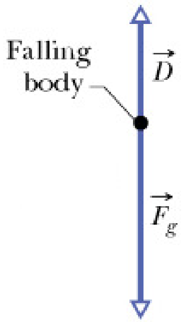
en use the relationship between a and v to set up an
uation to integrate

$$a = g - \frac{b}{m}v^n \Rightarrow \frac{dv}{dt} = -\frac{b}{m}\left(v^p - \frac{mg}{b}\right) \Rightarrow \int_{v_0}^{v} \frac{dv}{\left(v^p - \frac{mg}{b}\right)} = -\frac{b}{m}\int_0^t dt$$

  - The nature of the solution depends critically upon the power (p) chosen…

Drag Force and Terminal Speed

When an object moves through a fluid (gas or liquid) it experiences an opposing force known as "drag." Under certain conditions (the moving object must be blunt and must move fast so the flow of the liquid is turbulent) the magnitude of the drag force is given by the expression



$$D = \frac{1}{2} C \rho A v^2$$

Here C is a constant, A is the effective cross-sectional area of the moving object, ρ is the density of the surrounding fluid, and v is the object's speed. Consider an object of mass m that starts moving in air. Initially $D = 0$. As the object accelerates D increases and at a certain speed v_t, $D = mg$.

$$\frac{1}{2} C \rho A v_t^2 = mg \qquad v_t = \sqrt{\frac{2mg}{C \rho A}}$$

At this point the net force and thus the acceleration become zero and the object moves with constant speed v_t known as the **terminal speed**.

Activity - Air Resistance and Velocity

- **Assume a drag force of the form,**

$$D = \frac{1}{2} C \rho A v^2$$

- **Determine the exponent required for falling coffee filters at terminal speed**

$$\frac{1}{2} C \rho A v_t^2 = m g$$

  - Drop from ~2 m above floor
  - Measure time to fall from top of table
  - Assume that terminal velocity is reached
    - *Therefore $d = V_T t_{fall}$*
    - *Plot Terminal Velocity (cm/s) Vs. Weight (g*cm/s^2)*
    - *From slope and cross-sectional area determine C*

Activity - Determining the Exponent

- Assume a drag force of the form

$$F_{\text{air}} = K\, v^p$$

- Determine the power, p by plotting the logarithm of the force at v_T

$$\begin{aligned}\log(F_{\text{air}}) &= \log(K\, v^p) \\ &= \log(K) + \log(\, v^p) \\ &= \log(K) + p\log(\, v)\end{aligned}$$

log(mg) = p log (v_T) + log (K)

(y = m x + b)

The Dependence of Air Resistance on Velocity

The force of air resistance clearly depends on the velocity of an object moving through the air: the larger the speed, the larger the drag force. But what is the exact form of this relationship?

Your textbook suggests that under some circumstances, air resistance depends on the square of the velocity:

$$F_{\text{air}} = K\, v^2$$

However, some other sources suggest that at low speeds, the air resistance grows linearly with velocity:

$$F_{\text{air}} = K\, v$$

Your job today is to figure out which of these formulae more accurately fits the data from a simple experiment.

The experiment

- Create a set of objects with the same size and shape, but different mass, by stacking coffee filters: Try using stacks of 2, 4, 6, 8, 10 filters. Write your name and the number of filters on the inner bottom surface of each stack.
- Give the objects what we HOPE will be terminal velocity by having one team member stand on the third floor of the atrium and drop the stacks, one at a time. After a short acceleration, each one will (we hope) reach a constant speed for the majority of its fall.
- Have a second team member stand at the bottom of the atrium and measure the time it takes for each stack to fall from the **level of the carpet on the first floor** to the bottom of the atrium.
- Make two trials for each stack.
- Calculate the speed of each stack during this final portion of its flight; the distance from atrium floor to first-floor carpet is 4.0 meters.

Compare your results to those of other groups. Did you find roughly the same speed for a stack with the same number of filters?

The model

When an object has reached terminal velocity, the downward pull of gravity exactly balances the upward push of air resistance:

$$F_{\text{air}} - mg = 0 \qquad \Rightarrow \qquad F_{\text{air}} = mg$$

That means that you can calculate the force of air resistance easily, if you know the mass of the falling filters.

Now, your job is to determine which of these relationships between the force of air resistance and velocity is a better fit to your measurements.

$$F_{\text{air}} = K\, v$$