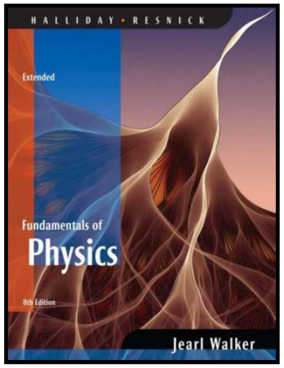
Workshop Physics 1017 - 311

University Physics I



Week 7 : Day 3

Conservation of Mechanical Energy

Conservation of Mechanical Energy :

Mechanical energy of a system is defined as the sum of potential and kinetic energies $E_{\text{mech}} = K + U$. We assume that the system is isolated, i.e., no external forces change the energy of the system. We also assume that all the forces in the system are conservative. When an internal force does work W on an object

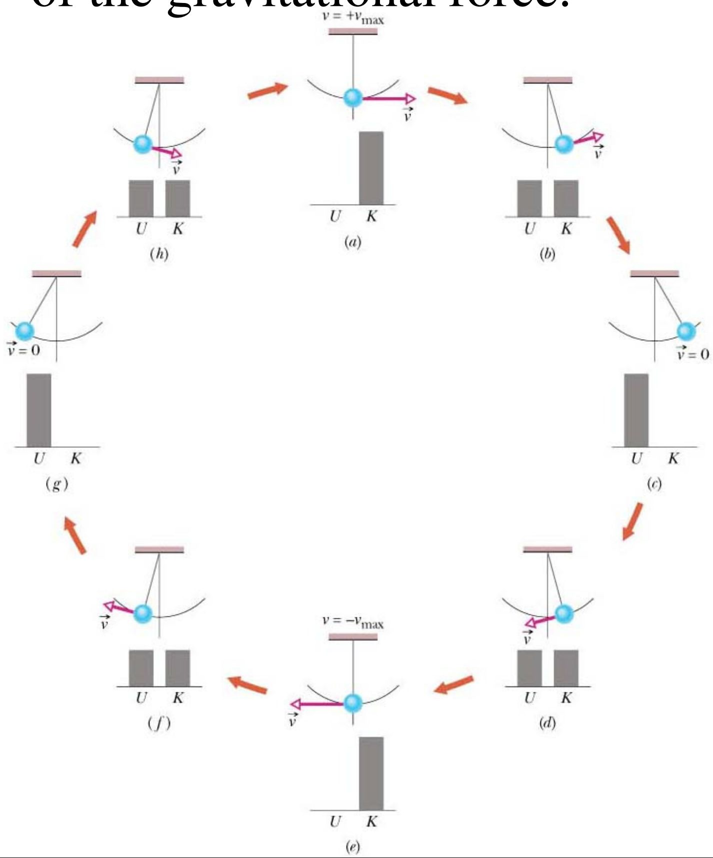
$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

For an isolated system in which the forces are a mixture of conservative and nonconservative forces the principle takes the following form:

$$\Delta E_{\text{mech}} = W_{nc}$$

Conservation of Mechanical Energy – An Example

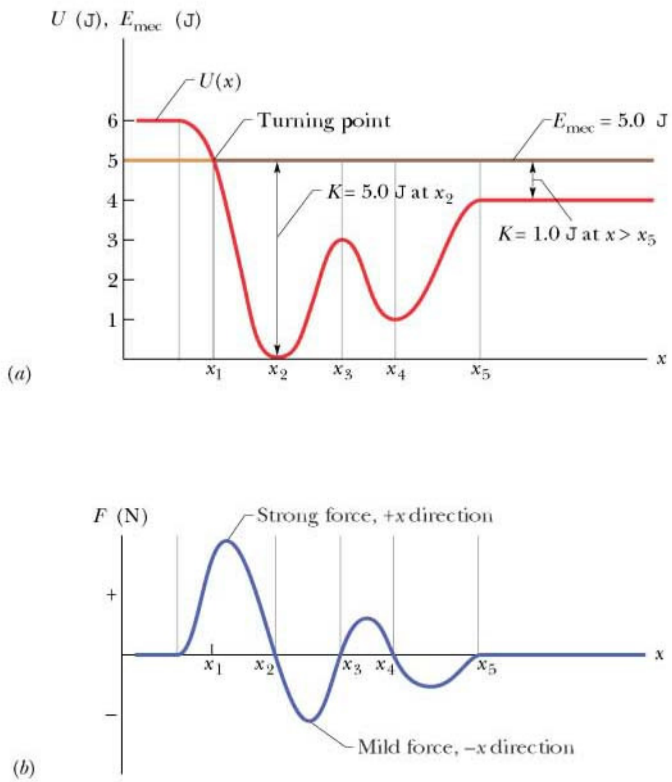
An example of the principle of conservation of mechanical energy is given in the figure. It consists of a pendulum bob of mass *m* moving under the action of the gravitational force.



The total mechanical energy of the pendulum-Earth system remains constant. As the pendulum swings, the total energy *E* is transferred back and forth between kinetic energy *K* of the pendulum and potential energy *U* of the pendulum-Earth system.

We assume that *U* is zero at the lowest point of the pendulum orbit. *K* is maximum in frames a and e (*U* is minimum there). *U* is maximum in frames c and g (*K* is minimum there).

Potential Energy Curves and Force



If we plot the potential energy U versus x for a force F that acts along the x-axis we can glean a wealth of information about the motion of a particle on which F is acting. The first parameter that we can determine is the force $F(x)$ using the equation

$$F(x) = -\frac{dU(x)}{dx}$$

- At x_2 , x_3, and x_4 the slope of the $U(x)$ vs. x curve is zero, thus $F = 0$.
- The slope dU/dx between x_3 and x_4 is negative; thus $F > 0$ for the this interval.
- The slope dU/dx between x_2 and x_3 is positive; thus $F < 0$ for the same interval.

Turning Points and Energy

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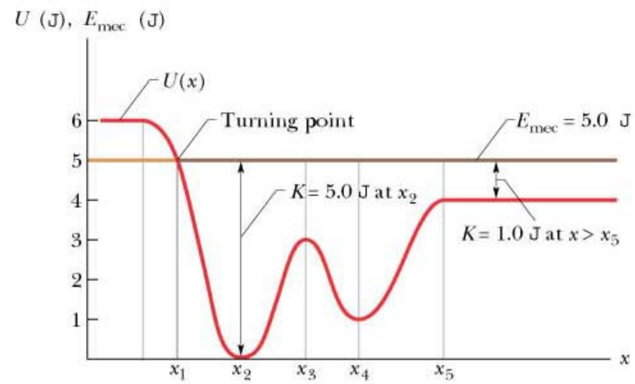
The total mechanical energy is $E_{mec} = K(x) + U(x)$. This energy is constant (equal to 5 J in the figure) and is thus represented by a horizontal line. We can solve this equation for $K(x)$ and get $K(x) = E_{mec} - U(x)$. At any point x on the x-axis we can read the value of $U(x)$. Then we can solve the equation above and determine K.



From the definition of $K = \dfrac{mv^2}{2}$ the kinetic energy cannot be negative.

This property of K allows us to determine in which regions of the x-axis motion is allowed. $K(x) = K(x) = E_{mec} - U(x)$

If $K > 0 \rightarrow E_{mec} - U(x) > 0 \rightarrow U(x) < E_{mec}$. Motion is allowed.

If $K < 0 \rightarrow E_{mec} - U(x) < 0 \rightarrow U(x) > E_{mec}$. Motion is forbidden.

The points at which $E_{mec} = U(x)$ are known as turning points for the motion. For example, x_1 is the turning point for the U versus x plot above. At the turning point, $K = 0$.

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$

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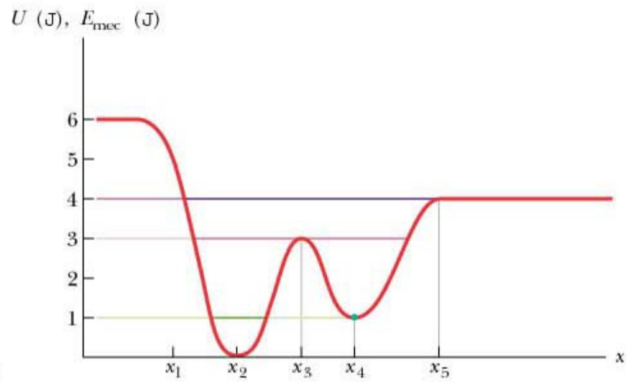
Equilibrium and Energy



Given the $U(x)$ versus x curve the turning points and the regions for which motion is allowed depend on the value of the mechanical energy E_{mec}.

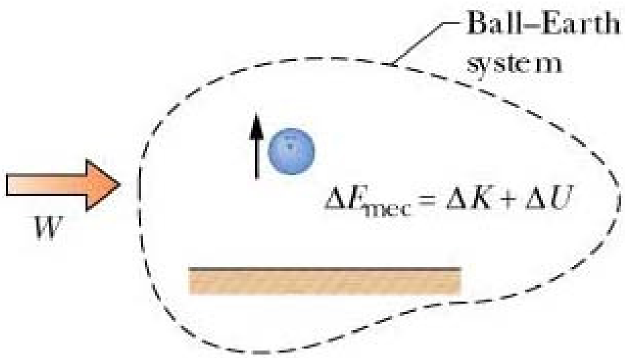
In the picture above consider the situation when $E_{mec} = 4$ J (purple line). The turning points ($E_{mec} = U$) occur at x_1 and $x > x_5$. Motion is allowed for $x > x_1$. If we reduce E_{mec} to 3 J or 1 J the turning points and regions of allowed motion change.

Equilibrium Points: A position at which the slope $dU/dx = 0$ and thus $F = 0$ is called an equilibrium point. A region for which $F = 0$ such as the region $x > x_5$ is called a region of **neutral equilibrium**. If we set $E_{mec} = 4$ J, the kinetic energy $K = 0$ and any particle moving under the influence of U will be stationary at any point with $x > x_5$.

Minima in the U versus x curve are positions of **stable equilibrium**.

Maxima in the U versus x curve are positions of **unstable equilibrium.**

Work Done on a System by an External Force



Up to this point we have considered only isolated systems in which no external forces were present. We will now consider a system in which there are forces external to the system.

The system under study is a bowling ball being hurled by a player. The system consists of the ball and the Earth taken together. The force exerted on the ball by the player is an external force. In this case the mechanical energy E_{mec} of the system is not constant. Instead it changes by an amount equal to the work W done by the external force according to the equation

$$W = \Delta E_{mec} = \Delta K + \Delta U$$

Activity – Energy Problems

- Work problems
  - Circular Motion
  - Spring Motion

Energy Problems – Circular Motion

1. A ball of mass m is attached to a string of length L. When the ball is at its lowest point it is given a velocity v_0, and it just barely makes a complete revolution. What must be the value of v_0? Answer in terms of m, g, and L. [Hint: you will need to use both the ideas of energy and the ideas of Newton here.]

2. A ball of mass m is attached to a string of length L. At the lowest point the ball has a speed v_0. For the moment assume that the ball is moving in a vertical circle.

(a) Find the speed of the ball as a function of the angle from the vertical.

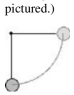
(b) Find the tension as a function of the angle.

(c) Use the result of part (b) to determine the minimum speed v_0^{min} that allows the ball to complete the circle.

(d) Suppose that $v_0 < v_0^{min}$, specifically $v_0 = 0.8\, v_0^{min}$. At what angle will the ball leave the circle?

(e) After the ball leaves the circle, sketch the path of the ball until the string again becomes taut.

3. A foam ball of mass m is attached to a very light rod of length L (to center of ball). The rod is held horizontally, and then the combination is released. When the ball reached the lowest point, the tension in the rod is $T = 2.5\, m\, g$. How much work was done by air resistance? The answer is in symbols, and only m, L, g and constants are allowed in the answer. The radius of the ball is small compared to L (not as pictured.)



4. Consider a hemisphere of ice of some unknown radius R. Your friend, of unknown mass sits at the top of the hemisphere, and a small gust of wind starts him (her) moving. At what angle, measured from the vertical, does your friend fly off the ice?

BONUS question: Where do they land?

Energy Problems - Springs

1. You decide to take the plunge and go bungee jumping. Being a college student, you do this on the cheap, and buy a bungee cord with a spring constant of $k = 500$ N/m and a length of $L = 8.0$ m. You choose a bridge where you tie the end of the cord so you can jump off. Assume that you step off the bridge with zero speed. How high must the bridge be so that you can do this fun jump more than once? (Being nervous, you want to be extra safe. What should you assume for air resistance?)

2. A spring is attached to a ceiling, and has a relaxed length of 25 cm. When a mass $m = 0.80$ kg is attached to the spring it stretches to an equilibrium length of $L_0 = 34$ cm.

(a) Find the spring constant of the spring.

(b) I lift the mass until the spring returns to its relaxed length, and then release it. When the mass returns to the equilibrium length, what is its speed?

(c) After I release the mass and it falls, what is the length of the spring when the mass reaches its lowest point?

3. A mass $m = 0.60$ kg is attached to a spring of constant $k = 75$ N/m and the system is hung vertically. Initially the mass is pulled down, stretching the spring by $x_0 = 12$ cm, and you throw the mass with an initial velocity $v_0 = 1.8$ m/s.

(a) Find the speed of the spring when it returns to its relaxed state.

(b) Find the lowest and highest points that the mass reaches.

4. A block of mass $m = 3.0$ kg is pressed up against a spring of constant $k = 240$ N/m. The block ***is not attached*** to the spring. It can slide along a board tilted at an angle $\theta = 37°$ to the horizontal. Initially the block is released from rest when the spring is compressed by $x_0 = 30$ cm.

a) Find the highest point that the block reaches.

5. A block of mass $m = 3.0$ kg is attached to a spring of constant $k = 240$ N/m. The block ***is attached*** to the spring. It can slide along a board tilted at an angle $\theta = 37°$ to the horizontal. Initially the block is released from rest when the spring is compressed by $x_0 = 30$ cm.

(a) Find the speed of the block when the spring is relaxed again.

(b) Find the highest point that the block reaches.