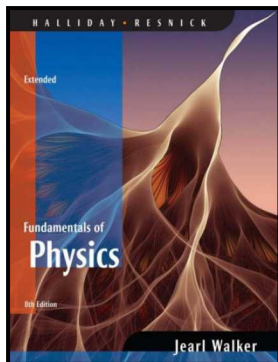


Workshop Physics

1017 - 311

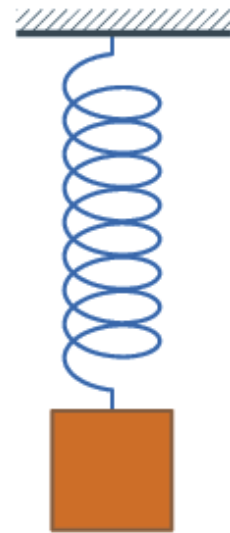
University Physics I



Week 8 : Day 1

Outline

- ❑ Preliminary Considerations
 - Static Equilibrium
 - Forces to consider
 - Newton's second law of motion
- ❑ The Simple Harmonic Oscillator (SHO)
 - A mathematical model
 - Parameters of SHO motion
 - Periodic and circular motion
- ❑ Analyzing the SHO System
 - Velocity and Acceleration in the SHO
 - Maximum and minimum velocity
 - Predicting the period of the motion
 - Making a velocity profile



Preliminary Considerations

□ Static Equilibrium

- A state w/o motion
- No *net* force on mass

□ Forces to Consider

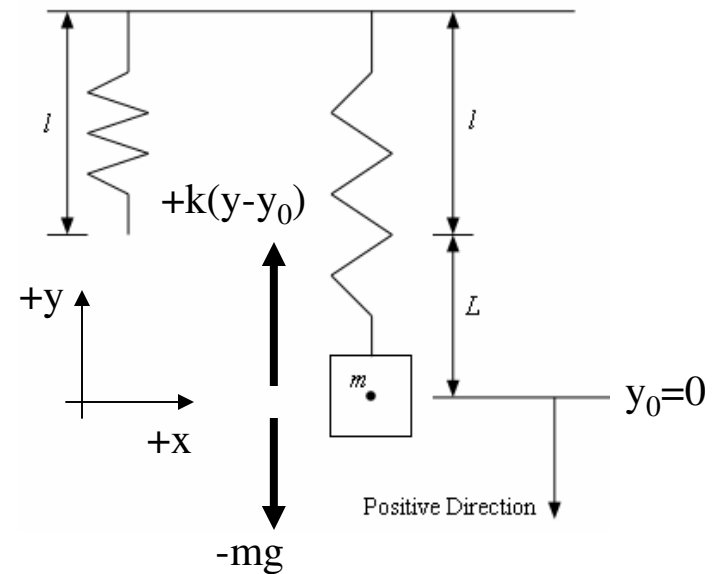
- Gravity - acts “downward”
- Spring force - acts “upward”
- Friction – non present here

□ Newton’s Second Law of Motion

- The *net* force is proportional to the acceleration:

$$\vec{F} = m\vec{a} \Rightarrow \underbrace{-mg}_{\text{gravity}} + \underbrace{k(L+l-l)}_{\text{spring}} = m(0) \Rightarrow mg = kL$$

- The forces are all balanced at the equilibrium position



The SHO System

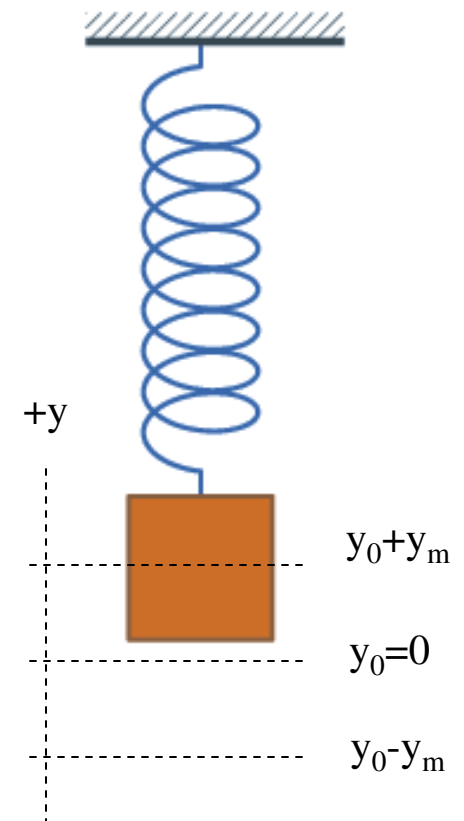
□ The Simple Harmonic Oscillator (SHO)

- Starts with a displacement upward
- Continues indefinitely w/o friction
- Repeats itself once every period

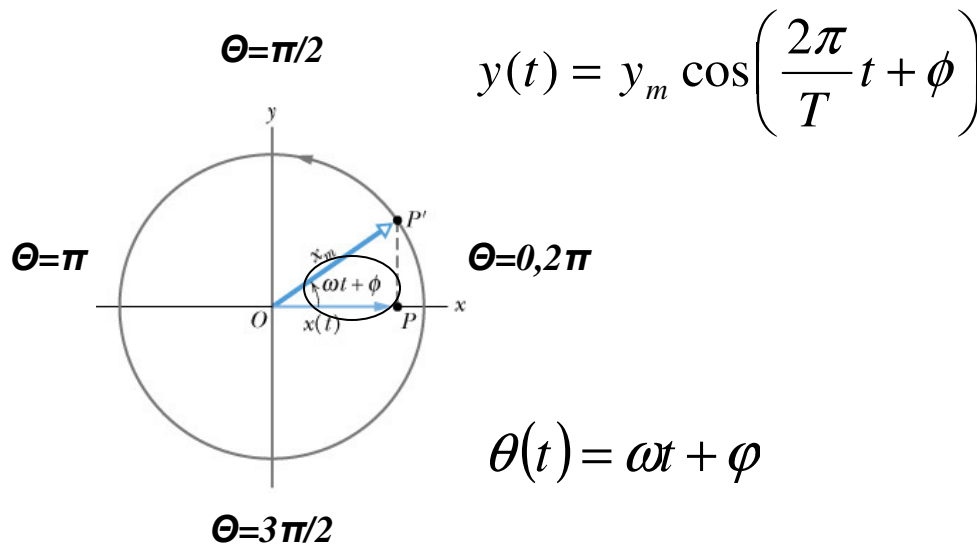
□ A Mathematical Model

- The motion is periodic – it repeats
 - *The time needed to complete one repetition is a period (symbol T: units s)*
 - *Model the SHO using a cosine function:*

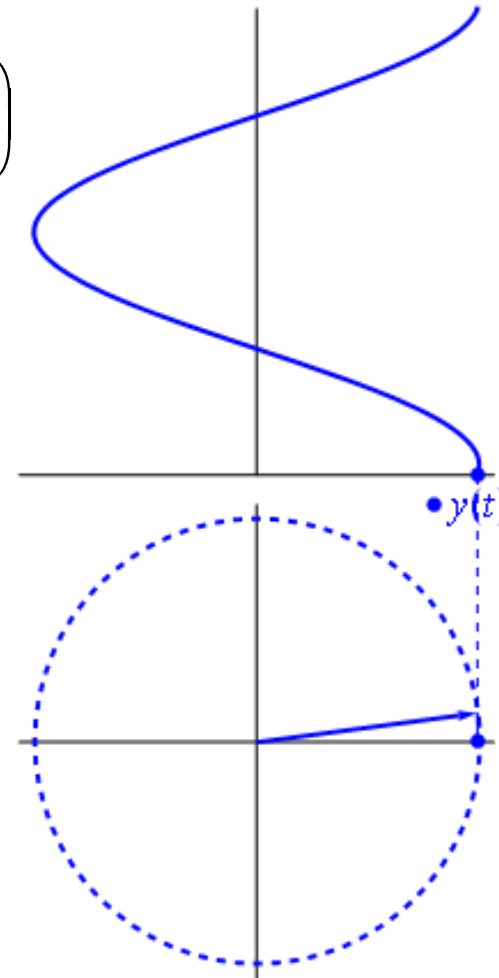
$$y(t) = y_m \cos\left(\frac{2\pi}{T}t + \phi\right)$$



Periodic and Circular Motion



$$y(t) = y_m \cos\left(\frac{2\pi}{T}t + \phi\right)$$



- The *amplitude* is the radius of the circle...
- The *period* is the time it takes to go around the circle one time...
- The *phase angle* determines the starting position...

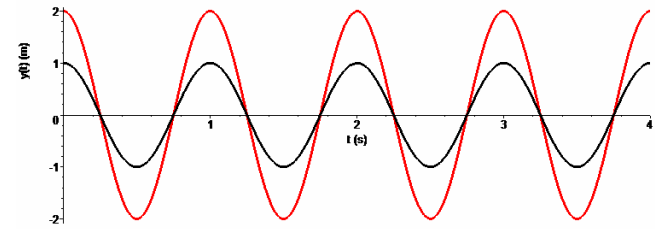
Parameters of SHO Motion

□ The Mathematical Model

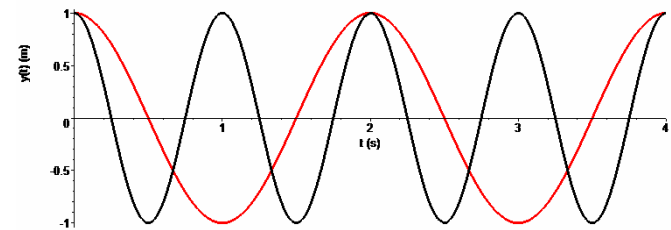
- Depends on the amplitude, y_m
- Depends on the period, T
- Depends on phase angle, ϕ

$$y(t) = \underbrace{y_m}_{\text{amplitude}} \cos \left(\underbrace{\frac{2\pi}{T}}_{\text{period}} t + \underbrace{\phi}_{\text{phase angle}} \right)$$

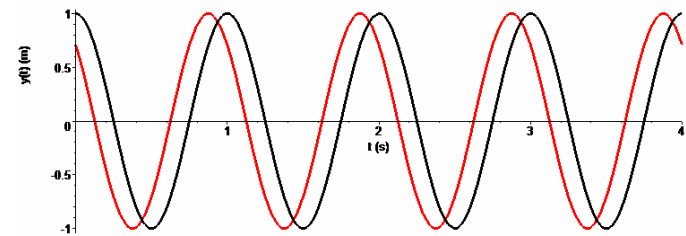
- Changing the mathematical parameters changes the model of the physical system



$$y_m = 1, y_m = 2 (T = 1, \Phi = 0)$$



$$T = 1, T = 2 (y_m = 1, \Phi = 0)$$



$$\Phi = 0, \Phi = \pi/4 (y_m = 1, T = 1)$$

Velocity of the SHO

□ Position

- Use the mathematical model provided

$$y(t) = y_m \cos\left(\frac{2\pi}{T}t + \phi\right)$$

□ Velocity

- Use the derivative of the position function

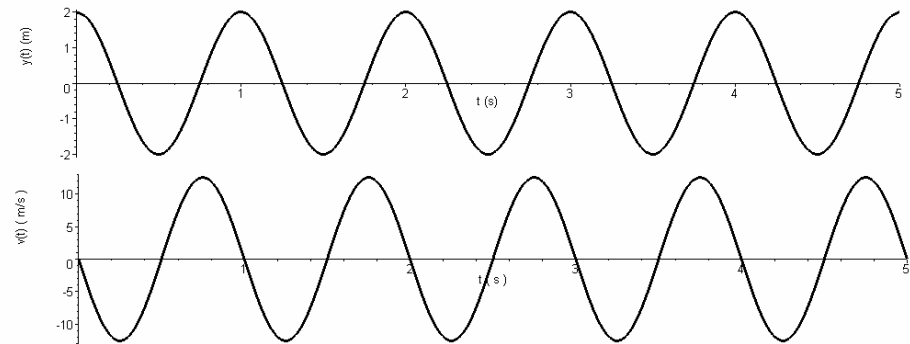
$$v(t) = \frac{d}{dt} y(t) = -\frac{2\pi}{T} y_m \sin\left(\frac{2\pi}{T}t + \phi\right)$$

□ Maximum Velocity

- The maximum velocity is the maximum of the velocity function and is given by:

$$v_{\max} = \frac{2\pi}{T} y_m = \frac{(2\pi y_m)}{T}$$

Position Vs. Time

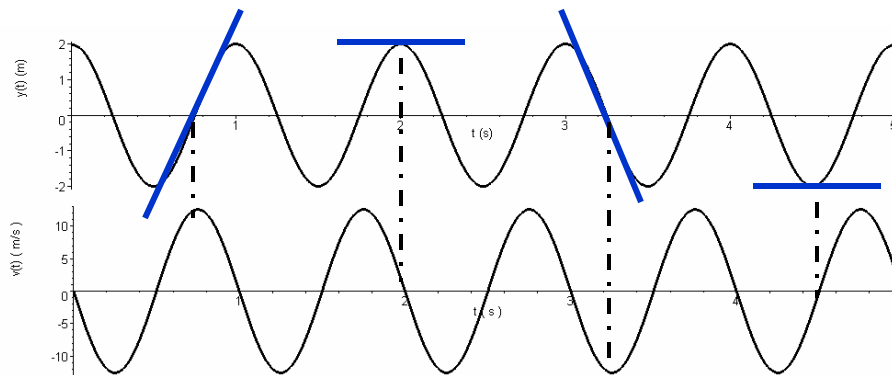


Velocity Vs. Time

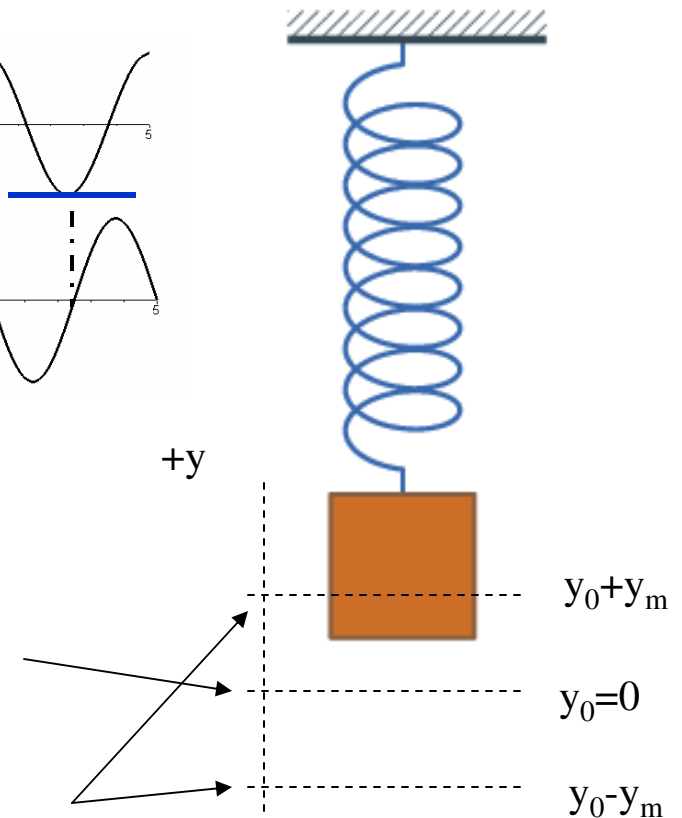
Maximum and Minimum Velocity

- Velocity is the Slope of the Position Curve

Look at the slope at various points on the graph



- The maximum velocity occurs when the displacement is zero.
 - Therefore the maximum velocity occurs at the equilibrium point.
- The minimum velocity occurs when the displacement is a maximum



Acceleration of the SHO

□ Velocity

- Use the derivative of the position function

$$v(t) = -\frac{2\pi}{T} y_m \sin\left(\frac{2\pi}{T}t + \phi\right)$$

□ Acceleration

- Use the derivative of the velocity function

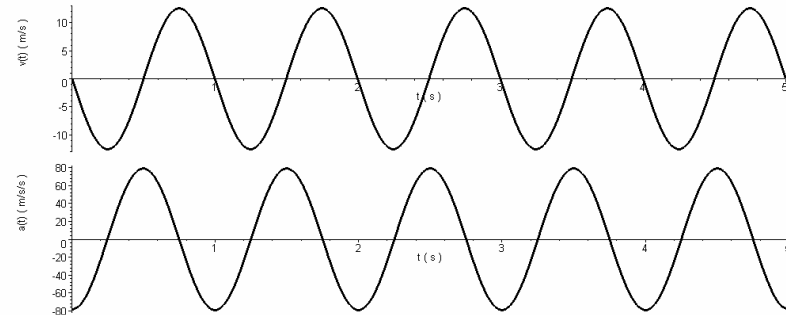
$$a(t) = \frac{d}{dt}v(t) = -\left(\frac{2\pi}{T}\right)^2 y_m \cos\left(\frac{2\pi}{T}t + \phi\right)$$

□ Maximum Acceleration

- The maximum acceleration is the maximum of the acceleration function and is given by:

$$a_{\max} = \left(\frac{2\pi}{T}\right)^2 y_m$$

Velocity Vs. Time



Acceleration Vs. Time

From Mathematics to Physics

□ What is the period of the motion?

- How is acceleration related to position?

$$\left. \begin{array}{l} y(t) = y_m \cos\left(\frac{2\pi}{T}t + \phi\right) \\ a(t) = -\left(\frac{2\pi}{T}\right)^2 y_m \cos\left(\frac{2\pi}{T}t + \phi\right) \end{array} \right\} \Rightarrow a(t) = -\left(\frac{2\pi}{T}\right)^2 y(t)$$

- Use Newton's second law to compare the physics to the math...

$$\left. \begin{array}{l} ma(t) = -ky(t) \\ -m\left(\frac{2\pi}{T}\right)^2 y(t) = -ky(t) \end{array} \right\} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{k}{m} \Rightarrow T = 2\pi\sqrt{m/k}$$

Making a Velocity Profile

□ Determine velocity at a given position

- Start with Newton's Second Law (again)
- Apply the Calculus that you know...

$$\Rightarrow m \frac{dv}{dt} = -ky$$

Use a chain rule
"trick" here to
eliminate time...

$$\frac{dv}{dt} = \frac{dv}{dy} \frac{dy}{dt} = v \frac{dv}{dy}$$

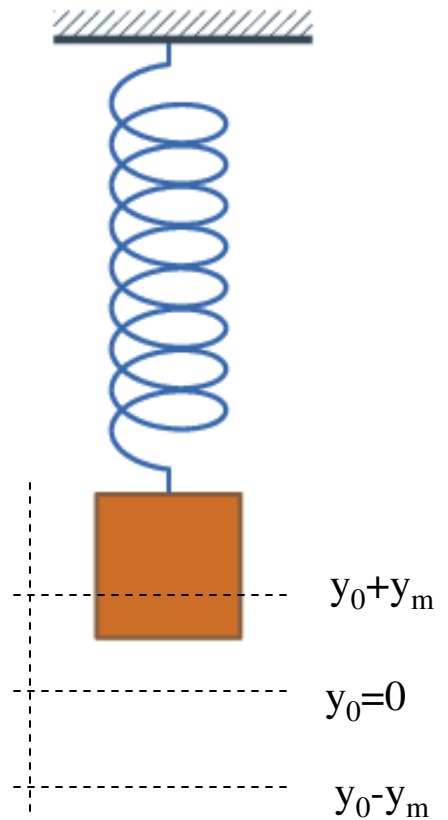
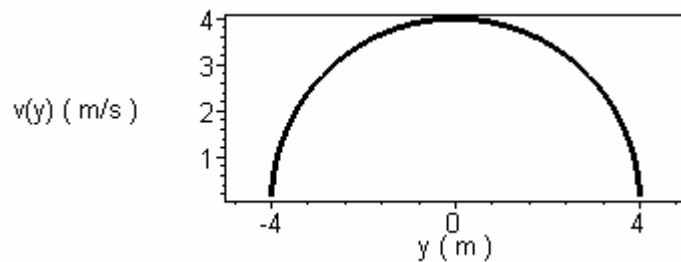
$$\Rightarrow mv dv = -ky dy$$

$$\Rightarrow m \int_0^v v dv = -k \int_{+y_m}^y y dy$$

$$\Rightarrow \frac{1}{2} mv^2 = \frac{1}{2} ky_m^2 - \frac{1}{2} ky^2$$

$$\Rightarrow v(y) = \sqrt{\frac{k}{m} [y_m^2 - y^2]}$$

Velocity Profile



Activity – Spring Oscillator

□ Setting up the experiment

- Set the slide switch on the force probe to 10 N. Place the motion sensor on the floor. Attach the force sensor to *CH 1*, and the motion sensor to *Dig/Sonic 1*.
- Go to "Student Shares" -> "University Physics" -> "Team Physics 311" -> "Lab Pro", Drag the file *Conservation_of_Energy* (note the underscores) onto your local desktop. Double-click on *Conservation_of_Energy* to start LabPro.
- Calibrate the force sensor.
- Choose one of the tiny springs. Attach the aluminum mass hanger and add a 50 g mass for a total hanging mass of 100 g. The mass should be about 15 or 20 cm below the bench top at equilibrium. Zero both sensors with the block at rest.
- Check your set-up by lifting the mass straight up about 5 cm and releasing it. Avoid side-to-side motion. What do you expect to see for position as a function of time? Collect data; do you see what you expected? If not, adjust the sensors until you get what you expect.
- You should see the position oscillate around 0 meters (to within about +/- 0.001 m), and the force oscillate around 0 Newtons (to within about +/- 0.01 N). The amplitude of the motion should be around 5–10 cm. Look carefully; if the centers of oscillation aren't zero, you may have to adjust the readings manually to force them to zero.

□ Analysis over a few cycles

- ONLY DO steps 1-9

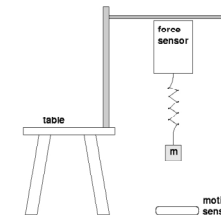
Conservation of Energy as a Mass on a Spring Oscillates

The goals

- Find the effective mass of the spring
- Find the spring constant of the spring
- Find the "phase difference" during oscillation between
 - a) force and displacement
 - b) force and acceleration
 - c) force and velocity
 - d) kinetic energy and spring potential energy
- Check to see if total energy is constant over several full cycles
- Find the "time constant" of energy loss over a period of three minutes

Optional:

- Investigate the effect of air resistance on energy loss.



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