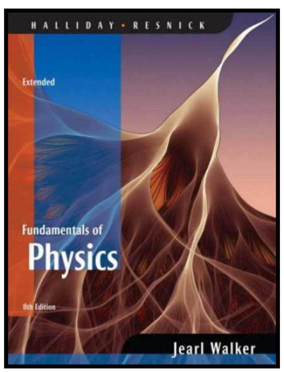
University Physics II

Week 1 : Day 1

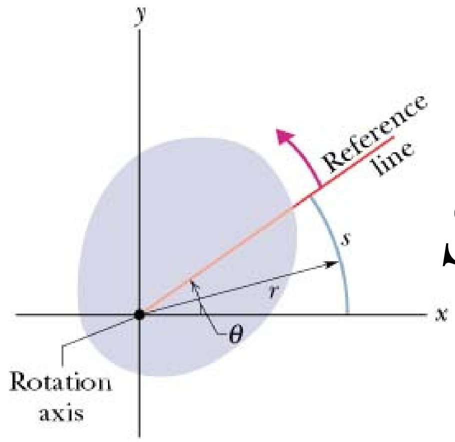


Outline

- **Rotational kinematics**
  - Angles and arcs
  - Reference lines and axes
  - Rotation direction and order
- **Rotational variables**
  - Equations of motion
  - Linear and angular relations

Angles and Arcs

- Angular motion may be quantified using the arc length, S



$$S = r\theta \quad \Rightarrow \theta = \frac{S}{r}$$

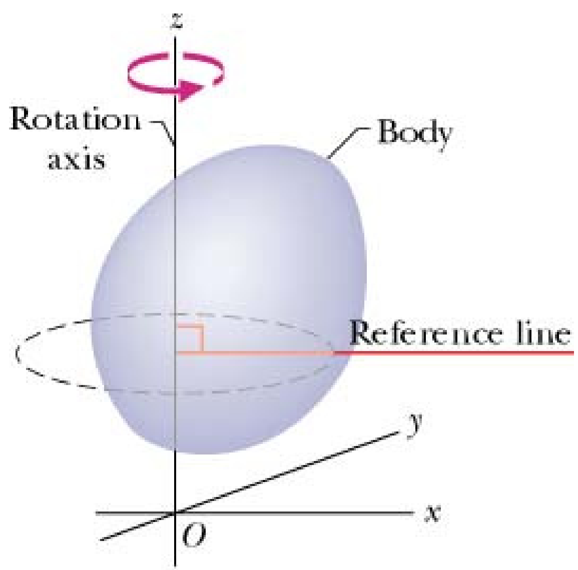
- The total circumference is given by,

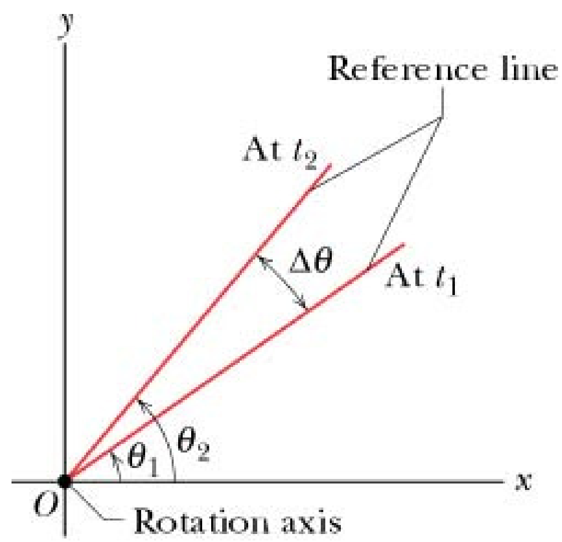
$$C = 2\pi r \quad \Rightarrow \frac{\theta}{2\pi} = \frac{S}{C}$$

*Note: θ is not a physical variable with units but rather a relative magnitude of a complete rotation…

References Lines and Axes

- The amount (or degree…) of a rotation is measured by a ***reference line*** from the ***rotation axis***.





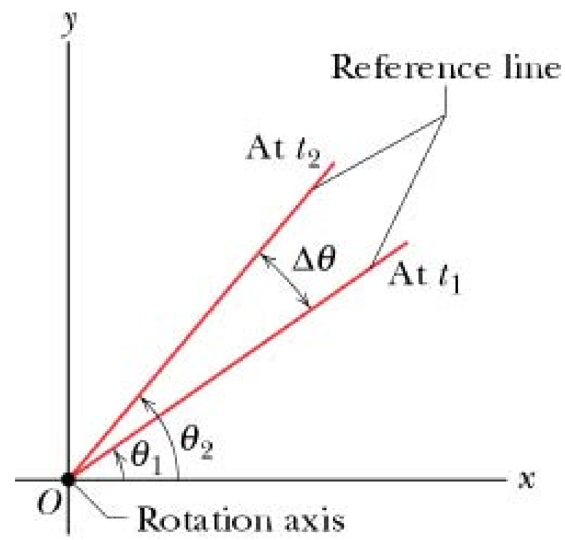
***Note: One revolution is equivalent to a rotation of 2π.**

Rotational Variables

- **Angular variables include**
  - Angular displacement, θ
  - Angular velocity, ω
  - Angular acceleration, α

$$\omega \equiv \frac{d\theta}{dt}$$

$$\alpha \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



“[variable]” = units of variable
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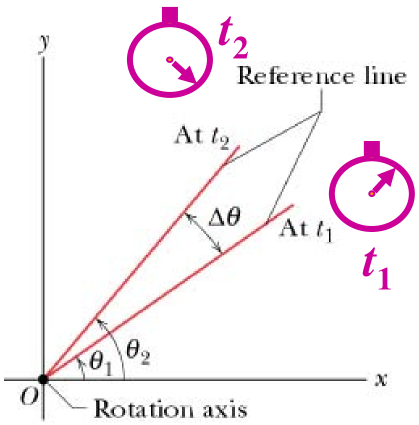
$$[\theta] = rad$$

$$[\omega] = \frac{rad}{s}$$

$$[\alpha] = \frac{rad}{s^2}$$

Angular Displacement and Velocity



In the picture we show the reference line at a time t_1 and at a later time t_2. Between t_1 and t_2 the body undergoes an angular displacement $\Delta\theta = \theta_2 - \theta_1$. All the points of the rigid body have the same angular displacement because they rotate locked together.

We define as average angular velocity for the time interval (t_1, t_2) the ratio

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}.$$

The SI unit for angular velocity is radians/second.

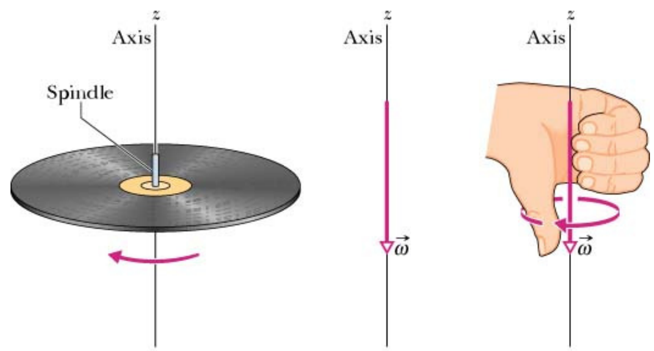
We define as the instantaneous angular velocity the limit of $\frac{\Delta\theta}{\Delta t}$ as $\Delta t \to 0$,

$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$. This is the definition of the first derivative with t: $\omega = \frac{d\theta}{dt}$

Algebraic sign of angular frequency: If a rigid body rotates counterclockwise (CCW), ω has a positive sign. If on the other hand the rotation is clockwise (CW), ω has a negative sign.

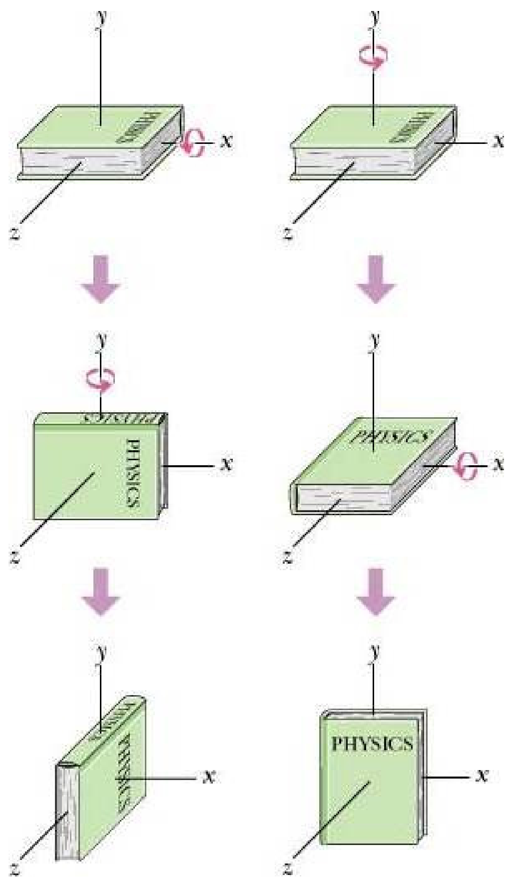
Rotational Direction and Order

- **Rotations as a vector**
  - Rotation is "around" direction axis.

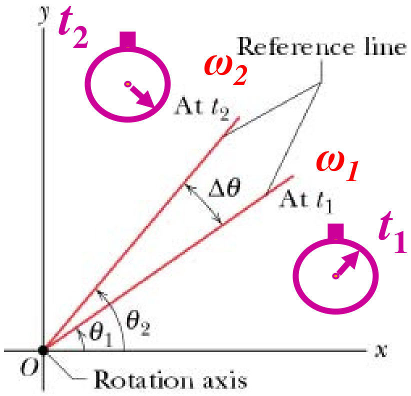


  - Rotation order matters!

$$XY \neq YX$$



Angular Acceleration



If the angular velocity of a rotating rigid object changes with time we can describe the time rate of change of ω by defining the angular aceleration.

In the figure we show the reference line at a time t_1 and at a later time t_2.
The angular velocity of the rotating body is equal to ω_1 at t_1 and ω_2 at t_2.
We define as average angular acceleration for the time interval (t_1, t_2) the ratio

$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$. The SI unit for angular velocity is radians/second^2.

We define as the instantaneous angular acceleration the limit of $\frac{\Delta\omega}{\Delta t}$ as $\Delta t \to 0$,

$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$. This is the definition of the first derivative with t : $\alpha = \frac{d\omega}{dt}$

RIT

Constant Acceleration Equations

When the angular acceleration α is constant we can derive simple expressions that give us the angular velocity ω and the angular position θ as a function of time.

Translational Motion		Rotational Motion	
x	$\leftrightarrow$	θ	
v	$\leftrightarrow$	ω	
a	$\leftrightarrow$	α	
$v = v_0 + at$	$\leftrightarrow$	$\omega = \omega_0 + \alpha t$	(eq. 1)
$x = x_o + v_0 t + \frac{at^2}{2}$	$\leftrightarrow$	$\theta = \theta + \omega_0 t + \frac{\alpha t^2}{2}$	(eq. 2)
$v^2 - v_0^2 = 2a\left(x - x_o\right)$	$\leftrightarrow$	$\omega^2 - \omega_0^2 = 2\alpha\left(\theta - \theta_0\right)$	(eq. 3)

Deriving Angular Relations

- **Start with the definition:**

$$\omega \equiv \frac{d\theta}{dt} \Rightarrow \omega dt = d\theta$$

- **Integrate *explicitly*: (specify limits)**

$$\int_0^t \omega dt = \int_{\theta_0}^{\theta} d\theta$$

$$\int_0^t (\omega_0 + \alpha t) dt = \theta - \theta_0$$

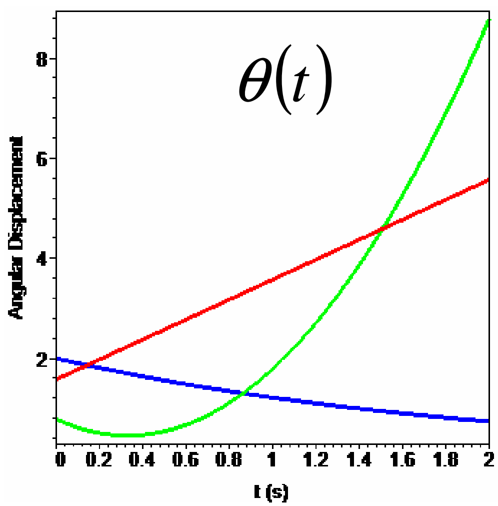
$$\Rightarrow \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

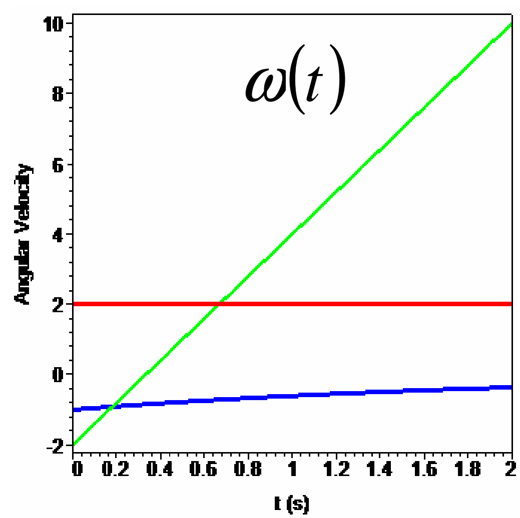
Constant Acceleration

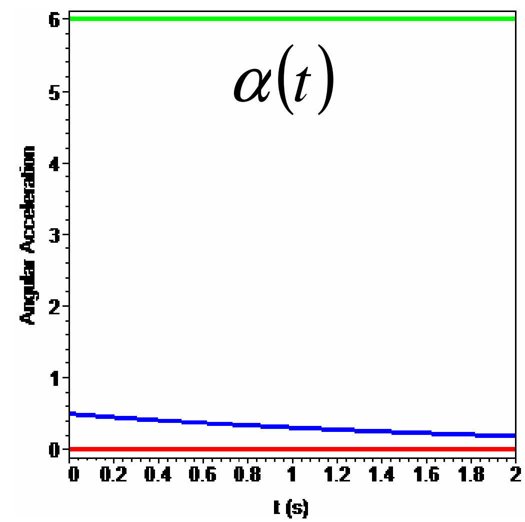
$$\omega(t) = \omega_0 + \alpha t$$

Angular Paths - Examples

- Consider three different angular paths:
  - **Path 1** $\theta(t) = 2t + \pi/2, \omega(t) = 2, \alpha(t) = 0$
  - **Path 2** $\theta(t) = 3t^2 - 2t + \pi/4, \omega(t) = 6t - 2, \alpha(t) = 6$
  - **Path 3** $\theta(t) = 2e^{-t/2}, \omega(t) = -e^{-t/2}, \alpha(t) = \frac{1}{2}e^{-t/2}$

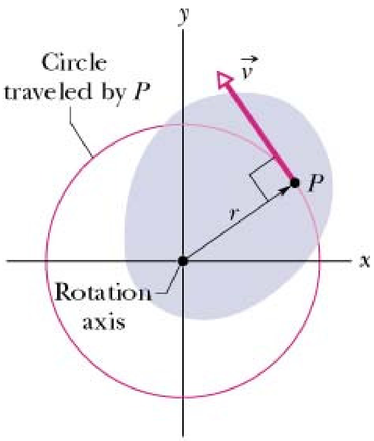






Angular and Linear Relations

- **Rotational motion about fixed axis**
  - Recall Uniform Circular Motion
  - Period of Motion is a Constant



velocity

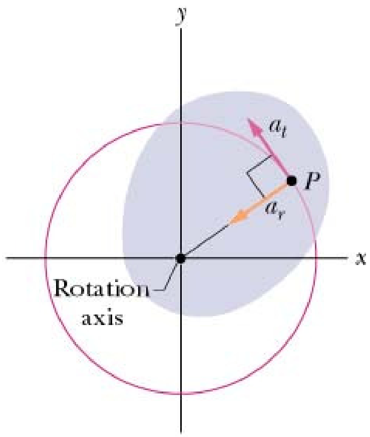
$$\frac{dS}{dt} = r\frac{d\theta}{dt} = r\omega$$

$$\Rightarrow v_t = r\omega$$

$$\Rightarrow v_t = \frac{2\pi r}{T} = \omega r$$

$$\Rightarrow a_r = \frac{v_t^2}{r} = r\omega^2$$

$$\Rightarrow \omega = \frac{2\pi}{T}$$



acceleration

$$a_t = \frac{dv_t}{dt} = r\frac{d\omega}{dt} = r\alpha$$

$$\Rightarrow a_t = r\alpha$$