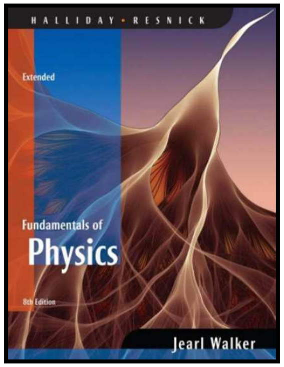
Workshop Physics 1017 - 312

University Physics II

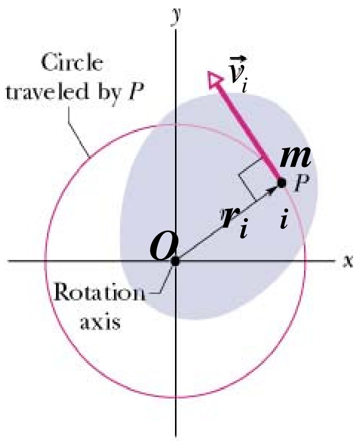


Week 1 : Day 3

Outline

- ***The Energy of Rotation***
  - *The Crab Nebula – Inertia to Spare!*
- **Torque and Inertia**
  - Moments of inertia
    - *Systems of Particles*
    - *Continuous Structures*
  - Calculating Moments
    - *Direct integration (1-D)*
    - *Table of Moments*
    - *Parallel Axis Theorem*

Energy of Rotation



Consider the rotating rigid body shown in the figure.
We divide the body into parts of masses $m_1, m_2, m_3, ..., m_i, ...$
The part (or "element") at P has an index i and mass m_i.
The kinetic energy of rotation is the sum of the kinetic energies of the parts: $K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + ...$

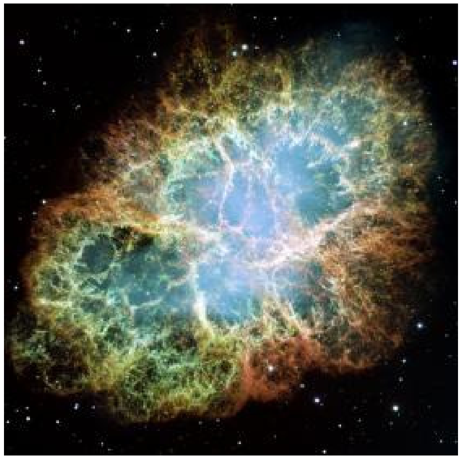
$K = \sum_i \frac{1}{2}m_iv_i^2$ The speed of the ith element $v_i = \omega r_i \rightarrow K = \sum_i \frac{1}{2}m_i(\omega r_i)^2$.

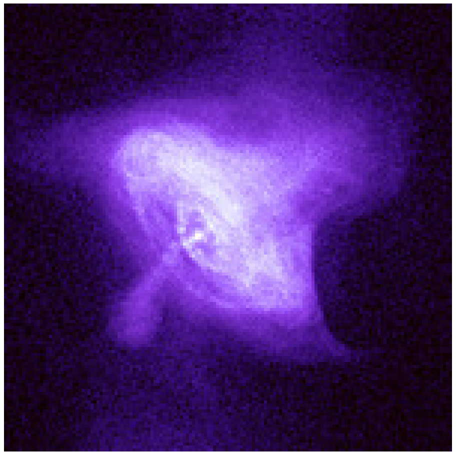
$K = \frac{1}{2}\left(\sum_i m_ir_i^2\right)\omega^2 = \frac{1}{2}I\omega^2$ The term $I = \sum_i m_ir_i^2$ is known as rotational inertia or moment of inertia about the axis of rotation. The axis of

$$\Rightarrow \quad K = \frac{1}{2}I\omega^2$$

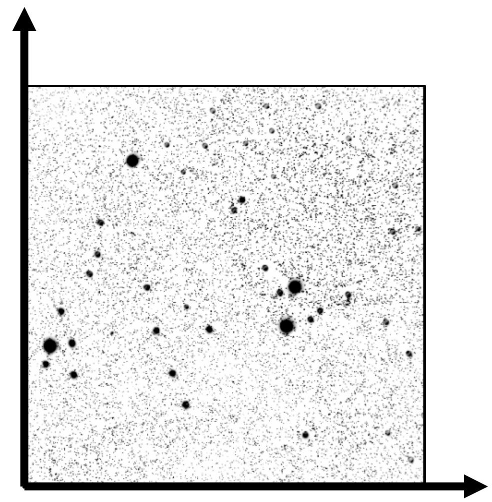
The Crab Nebula

- **The Crab Nebula**
  - Remnant of a star that exploded in 1054 A.D.
    - *Spans about 10 light-years*
    - *Located 6,000 light years away in the constellation of Taurus*
  - A neutron star is formed by the extreme conditions created in the supernova
    - *Core rotates about 30 times each second*
    - *Dense ball of neutrons that is twelve miles in diameter*





Moment of Inertia



Consider an arrangement of many particles (stars) as shown in the figure…

(The mass of a star is proportional to its' light intensity…)

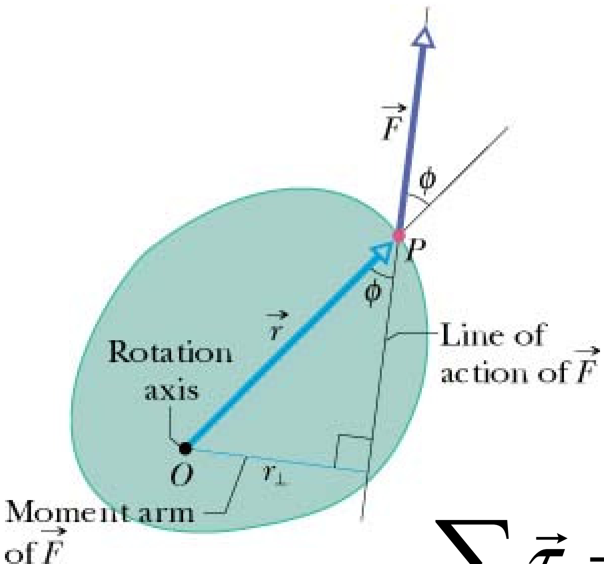
$K = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$ The term $I = \sum_i m_i r_i^2$ is known as rotational inertia or moment of inertia about the axis of rotation. The axis of rotation must be specified because the value of I for a rigid body depends on its mass, its shape, as well as on the position of the rotation axis. The rotational inertia of an object describes how the mass is distributed about the rotation axis.

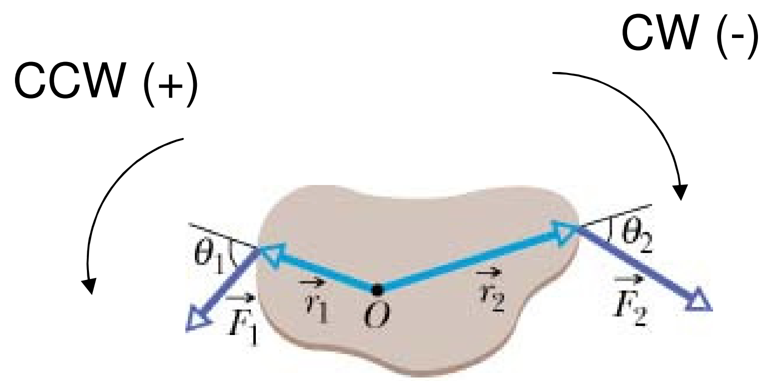
Discrete Particles → **Continuous Media**

$$I = \sum_i m_i r_i^2 \quad \Rightarrow \quad I = \int r^2 dm$$

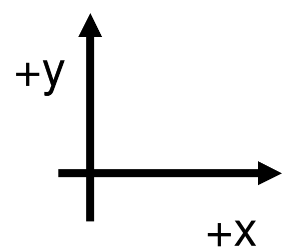
Torque and Inertia

- **Torque overcomes the inertia**
  - Force applied perpendicular to pivot arm
  - Parallel force does no work
    - *Provides no torque*





$$\sum \vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \ldots$$

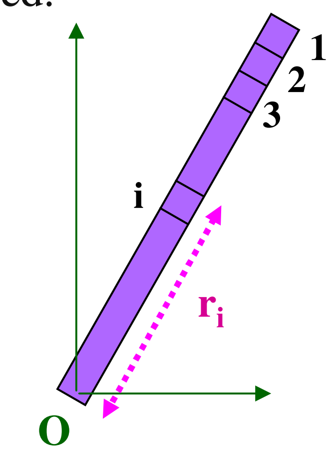


Newton's Second Law: Rotational Form

Consider the rod-like object shown in the figure, which can rotate about an axis through point O under the action of a net torque τ_{net}. We divide the body into parts or "elements" and label them. The elements have masses $m_1, m_2, m_3, \ldots, m_n$ and they are located at distances $r_1, r_2, r_3, \ldots, r_n$ from O. We apply Newton's second law for rotation to each element: $\tau_1 = I_1\alpha$ (eq. 1), $\tau_2 = I_2\alpha$ (eq. 2), $\tau_3 = I_3\alpha$ (eq. 3), etc. If we add all these equations we get $\tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n = \left(I_1 + I_2 + I_3 + \ldots + I_n\right)\alpha$. Here $I_i = m_i r_i^2$ is the rotational inertia of the ith element. The sum $\tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n$ is the net torque τ_{net} applied. The sum $I_1 + I_2 + I_3 + \ldots + I_n$ is the rotational inertia I of the body.

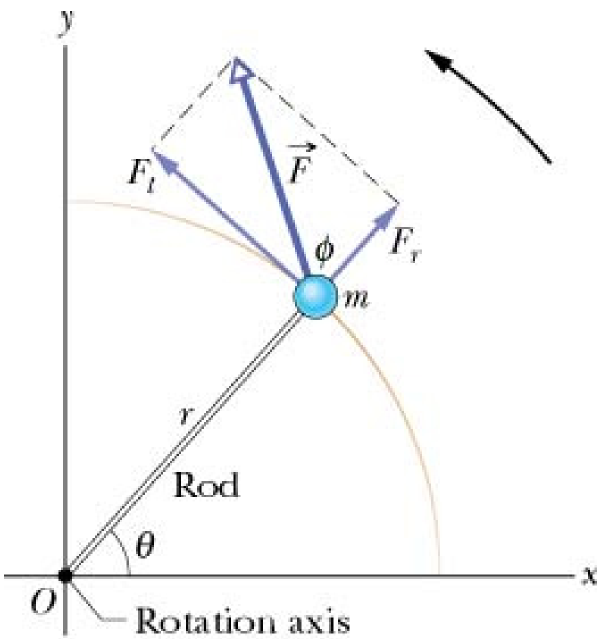
Thus we end up with the equation

$$\Rightarrow \vec{\tau}_{NET} = I\vec{\alpha}$$



Torque Calculations

- Consider a mass, m on the end of a massless rod of length, r:



CCW (+)

$$\tau = I_m \alpha$$

$$rF_t = \left(mr^2\right)\alpha$$

$$F_t = m(r\alpha)$$

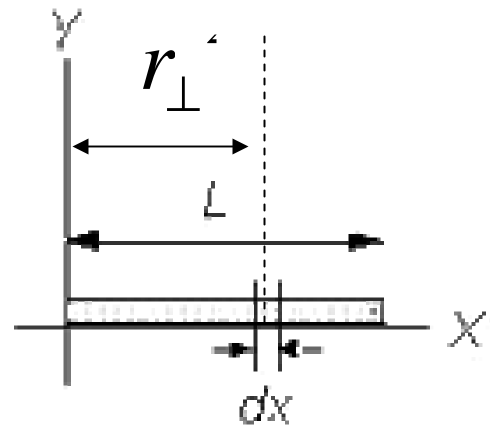
$$\Rightarrow F_t = ma_t$$

Calculating Moments - ID

- Start with the definition:

$$I = \int r_{\perp}^{2} dm$$

- Example: Thin Rod

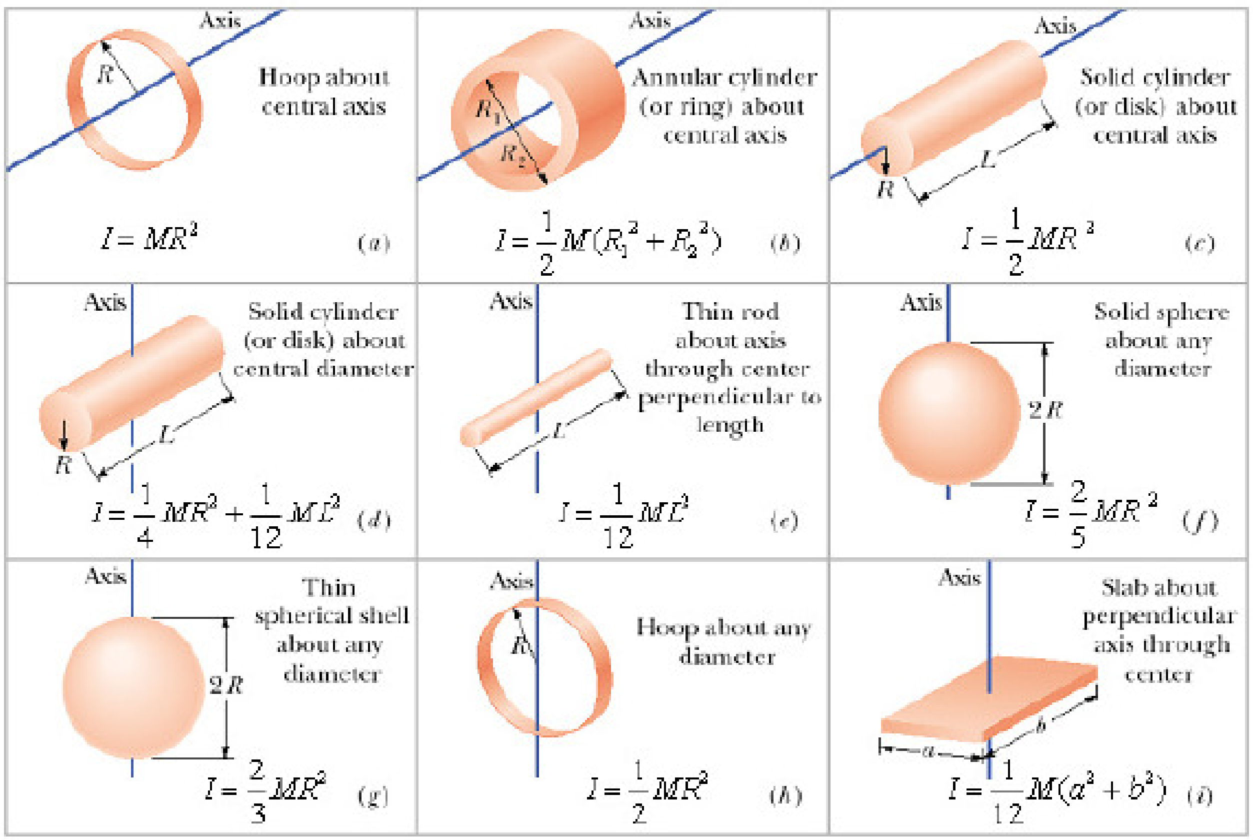


$$I_y = \int_0^L r_{\perp}^2 dm = \int_0^L x^2 \left(\underbrace{\frac{m}{L}}_{\lambda} \underbrace{dx}_{dl} \right) = \frac{m}{L} \frac{x^3}{3} \Bigg|_0^L = \frac{1}{3} mL^2$$

$$\lambda = \frac{dm}{dl}$$ ← Define a Linear Mass Density

Table of Moments

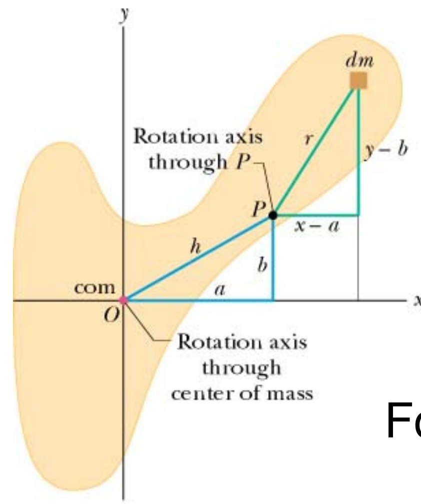
- Moments through COM for common objects



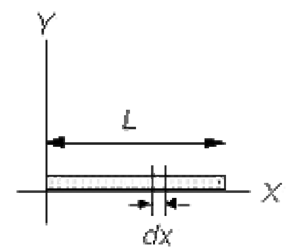
Parallel Axis Theorem

- **Inertia Calculations**
  - Moments are additive

$$I = I_{COM} + Mh^2$$

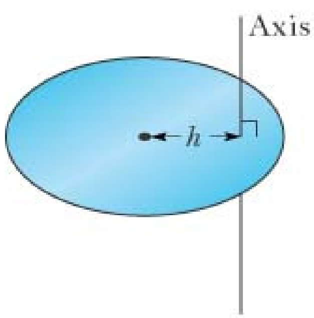


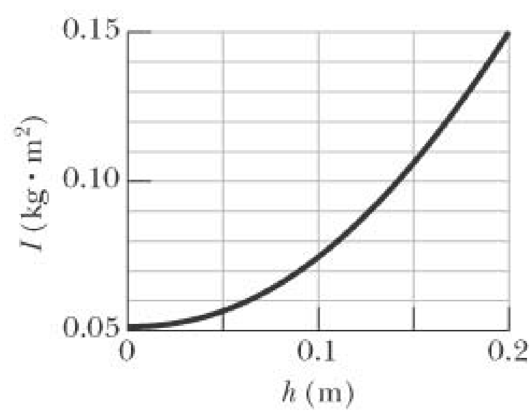
For Thin Rod:



$$I = I_{COM} + M\left(\frac{L}{2}\right)^2$$

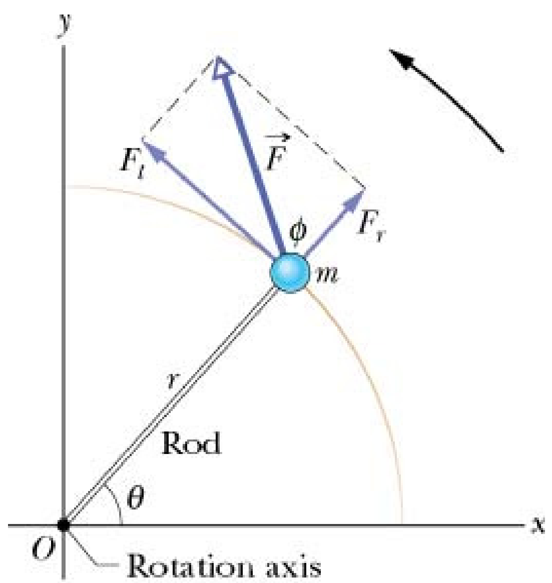
$$I = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$





Moment Calculations - Example

- **Consider massive rod with mass at end**
  - Torque calculation yields new result
  - Rod has mass M and length L
  - Moments of Inertia simply add – just like masses do



$$\Rightarrow \tau = \left(I_{mass} + I_{rod}\right)\alpha$$

$$LF_t = \left[mL^2 + \left(\overbrace{\tfrac{1}{12}ML^2}^{I_{COM}} + M\left(\tfrac{1}{2}L\right)^2 \right) \right]\alpha$$

$$LF_t = \left(mL^2 + \tfrac{1}{3}ML^2\right)\alpha$$

$$\Rightarrow F_t = \underbrace{\left(m + \tfrac{1}{3}M\right)}_{TotalMass} \underbrace{a_t}_{L\alpha}$$

Activity – Rotational Inertia

- Perform direct integration

Your Name (**Print**): ______________________ Date: ________
Group Members: ______________________
______________________ Group: ________

Rotational Inertia – Integration
(One Dimension)

The rotational inertia (or moment of inertia) for a continuous mass distribution is defined as

$$I = \int_{\substack{\text{rigid}\\ \text{body}}} dI = \int_{\substack{\text{rigid}\\ \text{body}}} r^2 dm$$

where r is the perpendicular distance of the mass dm from the axis of rotation.

The method to use is:

- draw a diagram showing the object and the axis of rotation
- choose a coordinate system that you think will be best to use
- pick a "slice" (or piece), dm, a perpendicular distance r from the axis of rotation; the piece should not be at the middle or the end of the object; show the distance r on your diagram
- write an expression for dm in terms of a small element of whatever coordinate(s) you have chosen
  $dm = \lambda\, dx$ for linear objects, where λ is the mass per length and dx is the length of dm;
- substitute for dI in the integral definition
- do the integral.

1. Calculate I_{com} for a long thin rod (for example, a meter stick) about an axis perpendicular to the rod, passing through its center of mass. Assume the mass is distributed uniformly along the rod. Express this moment of inertia as a function of its length L and mass M. As a check, compare your result to the table in the book.