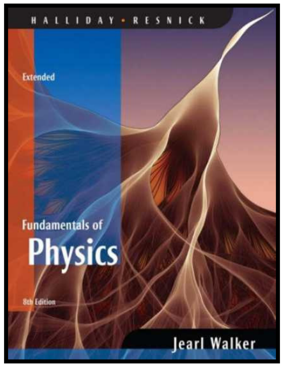
University Physics II

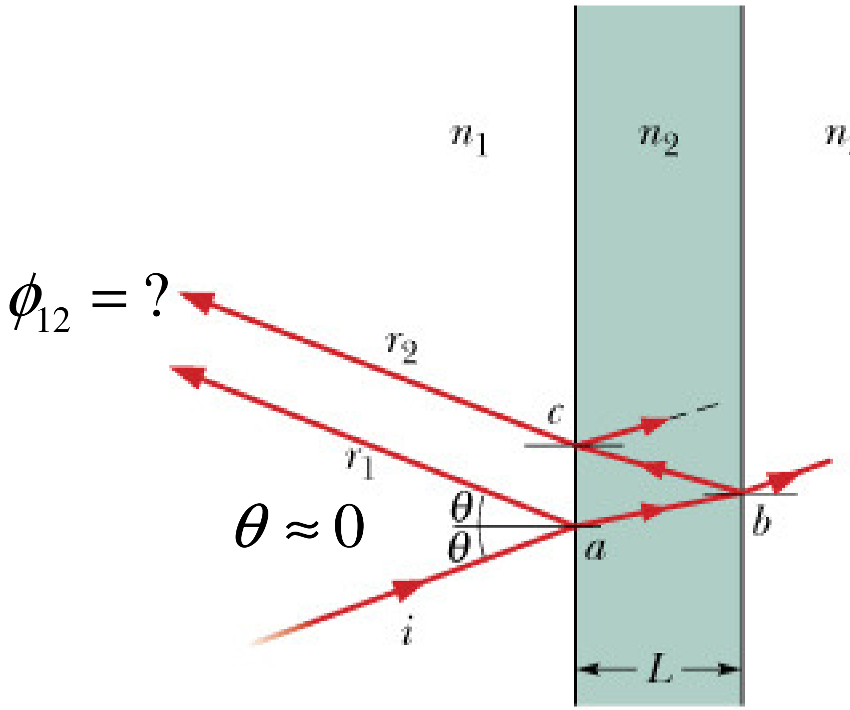


Week 10 : Day 1

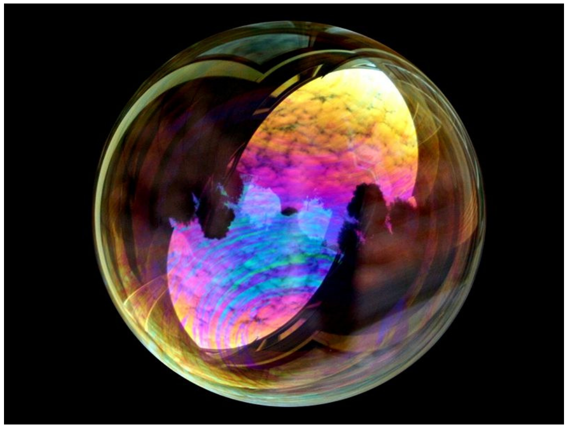
Outline

- **Thin Films**
  - Interference from thin films
  - Reflection and phase shift
  - Equations and problem solving tactics
- **Demonstrations**
  - Newton's rings…
- **Examples**
  - Paper currency
  - Water and oil
  - Non-reflective coatings
  - The Michelson interferometer
- **Activity**
  - Thin Film Applets

Interference from Thin Films

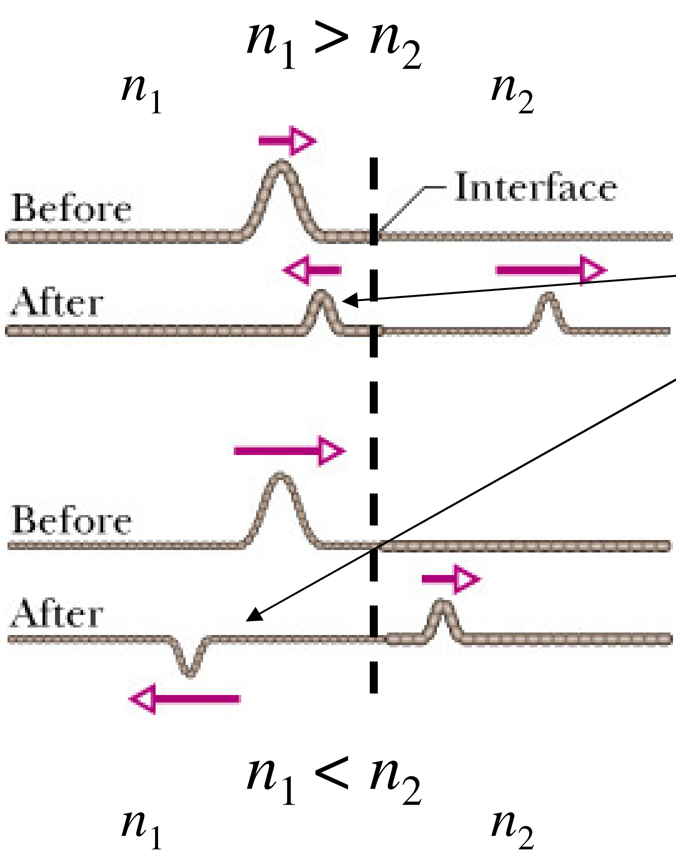


Interference between light waves is the reason that thin films, such as soap bubbles, show colorful patterns.



This is known as thin-film interference, because it is the interference of light waves reflecting off the top surface of a film with the waves reflecting from the bottom surface.

Reflection Phase Shifts



Reflection	Phase Shift
Off lower index	0
Off higher index	0.5 wavelength

Whenever light reflects off a surface of higher index of refraction, the wave is inverted. Peaks become troughs, and troughs become peaks. This is referred to as a 180° phase shift in the wave, but the easiest way to think of it is as an effective shift in the wave by half a wavelength…

Calculating Phase Differences

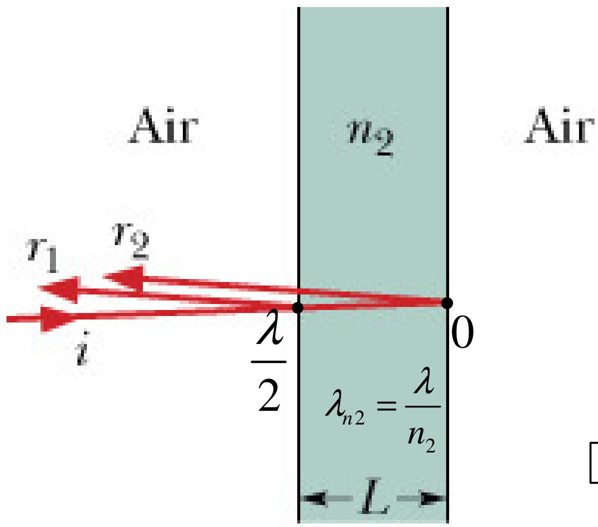
- **Two possible sources of phase difference:**
  - (1) The optical path length difference between rays: $L_{optical} = nL$

(optical path length) = (index of refraction)*(physical path length)

- *If the light is perpendicular to the film we need to use 2nL because it traverses the thickness of the film twice, once going down and once back up.*

  - (2) The phase introduced upon reflection: If light is in a material of lower index, and reflects from a boundary with a material of higher index, the phase change is π.
    - ***"Low off high, change of pi"***
    - ***"High off low, change of zero"***.
  - There is no 'in-between' in this case, it's either one or the other.

Equations for Thin-Film Interference



Three effects to consider:

1. Differences in reflection conditions.
2. Differences in path length traveled.
3. Differences in the refractive index.

One must use the wavelength in each medium (λ / n) to calculate the phase.

½ wavelength phase difference to difference in reflection of r_1 and r_2

$$2L = \frac{\text{odd number}}{2} \times \text{ wavelength} = \frac{\text{odd number}}{2} \times \lambda_{n2} \quad \text{(in-phase waves)}$$

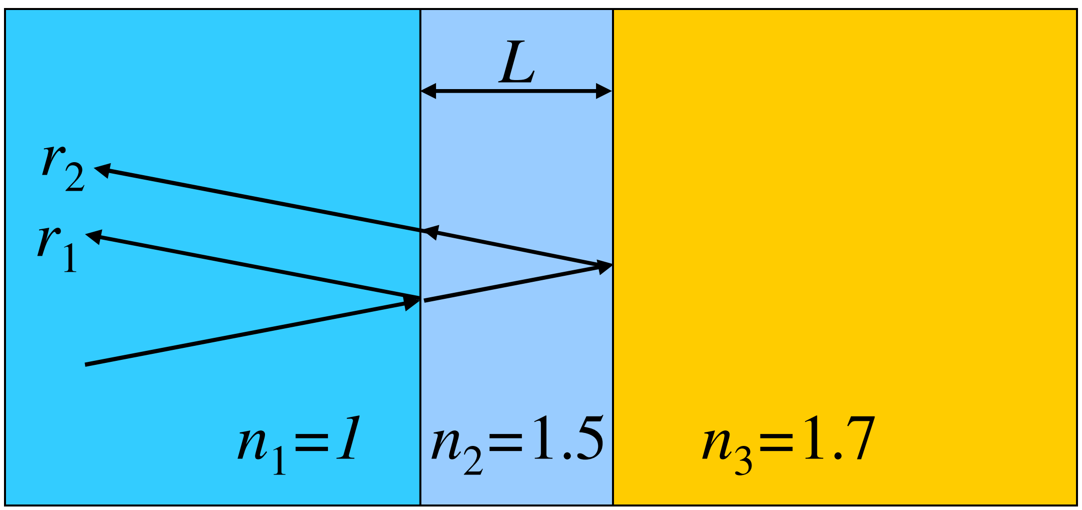
$$2L = \text{ integer } \times \text{ wavelength} = \text{integer} \times \lambda_{n2} \quad \text{(out-of-phase waves)}$$

Limitations of Thin-Film Equations

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \ldots \text{ (maxima-- bright film in air)}$$

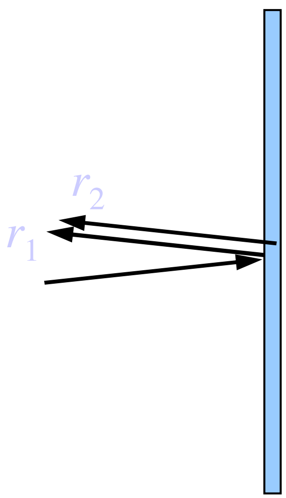
$$2L = m\frac{\lambda}{n_2} \quad \text{for } m = 0, 1, 2, \ldots \text{ (minima-- dark film in air)}$$

Note: These equations are for the special case of a higher index film flanked by air on both sides. For multilayer systems, this is not always the case and so these equations are not appropriate!



What happens to the above equations for the following multi-layer system?

Thin Film Limits



If L is much less than λ, for example $L < 0.1\lambda$, then phase difference due to the path difference $2L$ can be neglected.

Phase difference between r_1 and r_2 will always be ½ wavelength → destructive interference → film will appear dark when viewed from illuminated side.

Newton's Rings

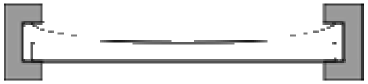
- **If a convex lens is placed on a glass slide and viewed in monochromatic light, a series of rings may be seen around the point of contact between the lens and the slide. These rings are known as *Newton's rings* - they arise from the interference of light reflected from the glass surfaces at the air film between the lens and the slide.**
  - *For dark fringes:* The thickness of the thin film, t , is given by $2t + \lambda/2 = (2m + 1)\lambda/2$ which simplifies to

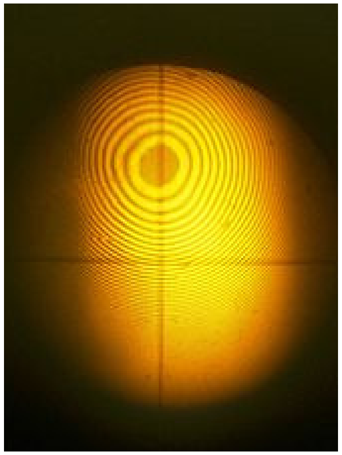
    $$2t = m\lambda$$

  - For *bright fringes:* The thickness of the film is given by $2t + \lambda/2 = m\lambda$
    which simplifies to

    $$2t = (m - 1/2)\lambda$$

The experimental setup: a convex lens is placed on top of a flat surface.



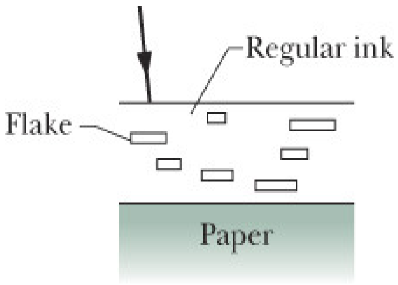


The radius of the N^{th} Newton's bright ring is given by

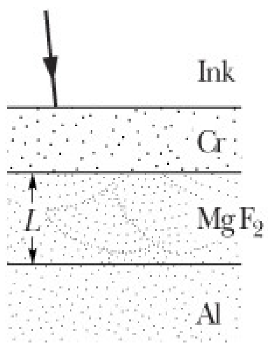
$$r_n = \left[\left(N - \frac{1}{2}\right)\lambda R\right]^{1/2},$$

Thin Films *on* & *in* Paper Currencies

Adapted from nrc-cnrc site at:
http://www.nrc-cnrc.gc.ca/aboutUs/nrc90/achievements/counterfeit_e.html



For the same path difference, different wavelengths (colors) of light will interfere differently. For example, $2L$ could be an integer number of wavelengths for red light but half-integer wavelengths for blue.



Furthermore, the path difference $2L$ will change when light strikes the surface at different angles, again changing the interference condition for the different wavelengths of light.

Take a close look at one of the newest Canadian $5 bills. If you hold it against a strong light you will see it contains a narrow strip of plastic that is embedded within the paper. This strip carries a special optical thin film coating that is exposed on the back of the bank note only in small rectangular patches where it touches the surface of the paper. The strip changes colour from gold to green when viewed at an increasing angle and it is inscribed with a repeating message "CAN 5."

Example: Oil on H_2O

- Step 1. Because oil has a higher index of refraction than air, the wave reflecting off the top surface of the film is shifted by half a wavelength.

$$\Delta_a = \lambda/2$$

- Step 2. Because water has a lower index of refraction than oil, the wave reflecting off the bottom surface of the film does not have a half-wavelength shift, but it does travel the extra distance of 2t.

$$\Delta_b = 2t$$

- Step 3. The relative shift is thus:

$$\Delta = \Delta_b - \Delta_a = 2t - \lambda/2$$

- Step 4. Now, is this constructive interference or destructive interference? Because the film looks red, there is constructive interference taking place for the red light.

For constructive interference : $\Delta = m\lambda$. In this case, $2t - \lambda/2 = m\lambda$

- Step 5. Moving all factors of the wavelength to the right side of the equation gives:

$$2t = (m + 1/2)\,\lambda \qquad (m = 0, 1, 2, 3, \ldots)$$

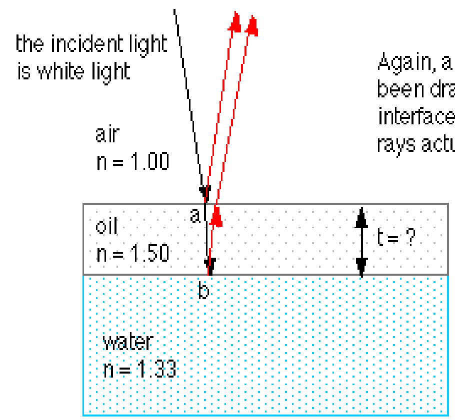
- Step 6. The wavelength in the equation above is the wavelength in the thin film. Writing the equation so this is obvious can be done in a couple of different ways:

$$2t = (m + 1/2)\,\lambda_2 \quad \text{or} \quad 2t = (m + 1/2)\,\lambda_{vac} / n_2$$

- Step 7. The equation can now be solved. In this situation, we are asked to find the minimum thickness of the film. This means choosing the minimum value of m, which in this case is m = 0. The question specified the wavelength of red light in vacuum, so:

$$2t_{min} = (1/2)\,\lambda_{vac} / n_2$$

This gives $t_{min} = \lambda_{vac} / 4n_2 = 636 / (4 \times 1.5) = 636 / 6 = 106$ nm

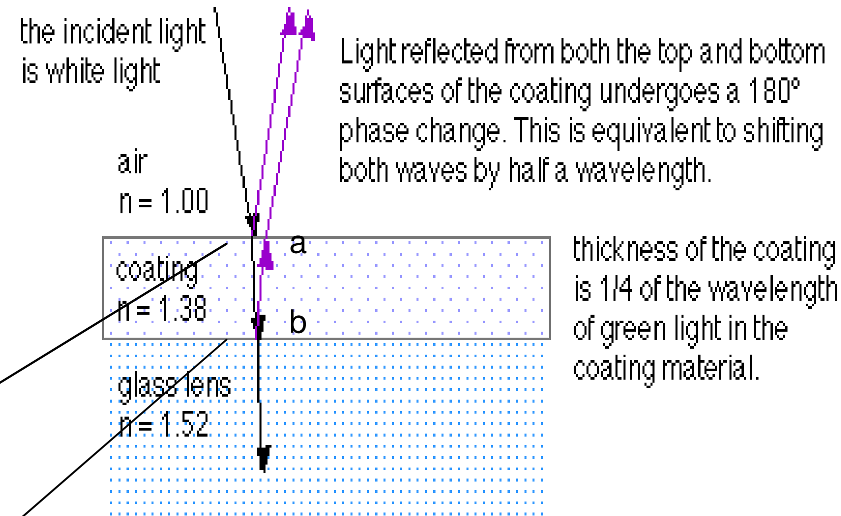


Again, although the rays have not been drawn perpendicular to the interface, that is just for clarity. The rays actually are perpendicular.

Note: that this looks like an equation for destructive interference! It isn't, because we used the condition for constructive interference in step 4. It looks like a destructive interference equation only because one reflected wave experienced a shift.

Example: Non-reflective Coatings

- **Destructive interference is exploited in making non-reflective coatings for lenses, etc.**
  - The coating material generally has an index of refraction less than that of glass, so both reflected waves have a $\lambda/2$ shift.
  - A film thickness of 1/4 the wavelength in the film results in destructive interference.

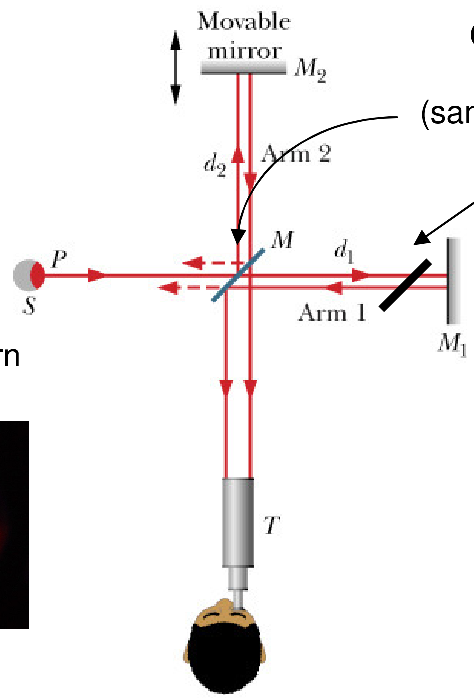


$\Delta_a = \lambda/2$ $\quad$ $\Delta_b = 2t + \lambda/2$ $\quad$ $\Delta = 2t = (m + 1/2)\,\lambda_{coating}$

$$t_{min} = \lambda_{coating} / 4 = \lambda_{vacuum} / (4\,n)$$

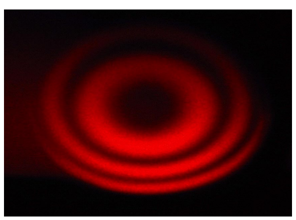
Michelson Interferometer

$$\Delta L = 2d_1 - 2d_2 \quad \text{(interferometer)} \Longrightarrow \Delta L_m = 2L \quad \text{(slab of material of thickness } L \text{ placed in front of } M_1\text{)}$$



Compensation Plate Inserted Here (same Thickness as Mirror)

Diffraction Pattern Observed Here



$$N_m = \frac{2L}{\lambda_m} = \frac{2Ln}{\lambda}$$

Number of Wavelengths in Thickness of Material

$$N_a = \frac{2L}{\lambda}$$

Number of Wavelengths in Same Thickness of Air

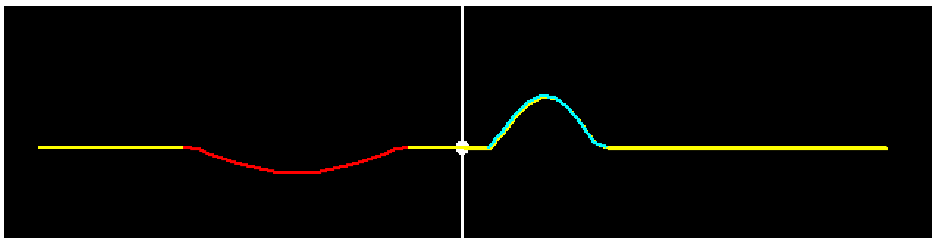
$$N_m - N_a = \frac{2Ln}{\lambda} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n-1)$$

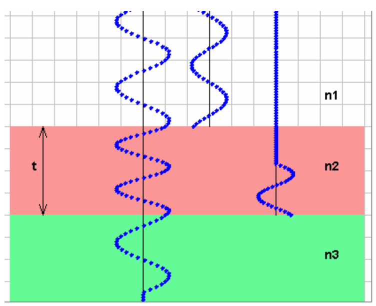
How it works: For each change in path by 1l, the interference pattern shifts by one fringe at *T*. By counting the fringe change, one determines N_m - N_a and can then solve for L in terms of l and n…

Activity – Thin Film Applets

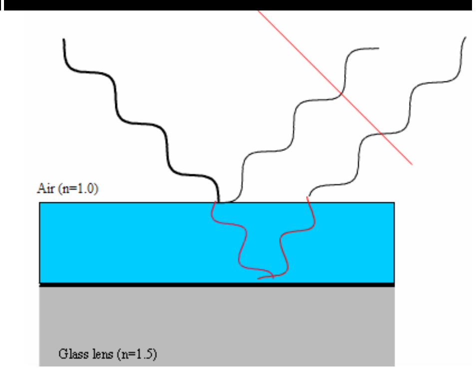
- ❑ **Simulate and learn!**

Mechanical Analogs





Index Variation



Reflective Coatings

Your Name (**Print**): ____________________ Date: ________
Group Members: ____________________
____________________ Group: ________

<u>Thin Films</u>

Key Point: In thin films there are <u>two</u> possible sources of phase difference:

(1) The optical path length difference between rays: $L_{optical} = nL$

(optical path length) = (index of refraction)*(physical path length)

If the light is perpendicular to the film we need to use $2nL$ because it traverses the thickness of the film twice, once going down and once back up.

(2) The phase introduced upon reflection: If light is in a material of lower index, and reflects from a boundary with a material of higher index, the phase change is π. **"Low off high, change of pi"**, and conversely, "**High off low, change of zero**". There is no 'in-between' in this case, it's either one or the other.

Let's quickly look at (2) first with the applet at:

`http://www.surendranath.org/Applets.html`

Click on the "Applet Menu" box in the upper left corner, then "waves" → "Transverse waves" → "reflection and transmission". Try the different cases available. It's using string, but the analogy to light waves is exactly the same. This is a lot safer using applets for this part than rigging up springs all over the room.

Now let's look at part (1) as well, including the difference in the optical path length between two routes. Suppose a piece of glass is coated with a thin layer of another material, and we shine light on it. The sources of the two rays of light are the reflection from the top layer, and the reflection from the boundary. Load the animation:

`http://webphysics.davidson.edu/physlet_resources/bu_semester2/c26_thinfilm.html`

This one is drawn with the two light rays off to the side so you can see what's going on a little more clearly. Click the "1 wavelength" box on the first ($n_1 < n_2$ and $n_2 > n_3$) choice.

For $n_1 < n_2$, what is the phase change off the top surface? ____________
For $n_2 > n_3$, what is the phase change off the bottom surface? ____________

(Check that what's going on in the applet agrees with the reflection song)