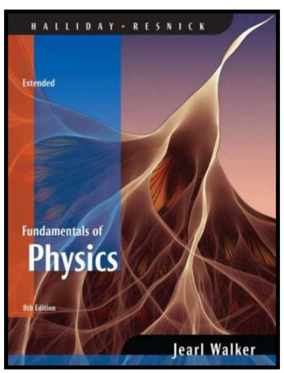
Workshop Physics 1017 - 312

University Physics II



Week 2 : Day 1

RIT

Outline

- **Direct Calculation of Inertia**
  - Integration methods
  - Integration examples
    - *One-dimensional (ID) Integration*
    - *Two-dimensional (2D) Integration*
    - *Three-dimensional (3D) Integration*

- **The Atwood Machine**
  - Free-Body Diagram
  - Solutions and Simulations
  - Activity – Rotational Inertia: Pre-Lab exercise

Integration Methods

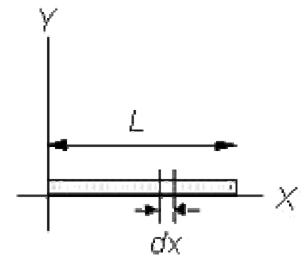
- Draw a <u>diagram</u> showing the object and the axis of rotation
- Choose an appropriate <u>coordinate system</u>
- Pick a "slice" (or piece), *dm*, and perpendicular distance *r*
  - Show the distance *r* on your diagram
- Write an expression for *dm* in terms of a small element
  - In 1-D problems $dm = \lambda\, dx$
  - In 2-D problems $dm = \sigma\, dxdy$
  - In 3-D problems $dm = \rho\, dxdydz$
- Perform the integral

$$I = \int r_{\perp}^{2}\, dm$$

Calculating Moments - ID

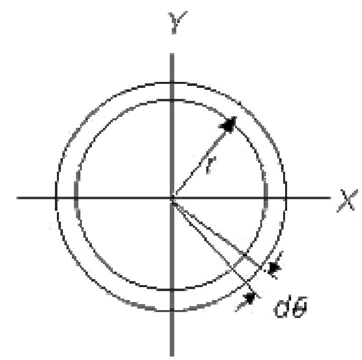
- Thin Rod

$$I_y = \int_0^L r_\perp^2 dm = \int_0^L x^2 \left(\underbrace{\frac{m}{L}}_{\lambda} \underbrace{dx}_{dl} \right) = \frac{m}{L} \frac{x^3}{3} \bigg|_0^L = \frac{1}{3} mL^2$$



- Circular Ring

$$I_z = \int_0^{2\pi} r_\perp^2 dm = \int_0^{2\pi} r^2 \left(\underbrace{\frac{m}{2\pi r}}_{\lambda} \underbrace{r d\theta}_{dl} \right) = \frac{mr^2}{2\pi} \int_0^{2\pi} d\theta = mr^2$$



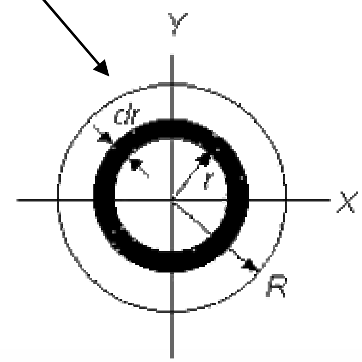
Calculating Moments - 2D

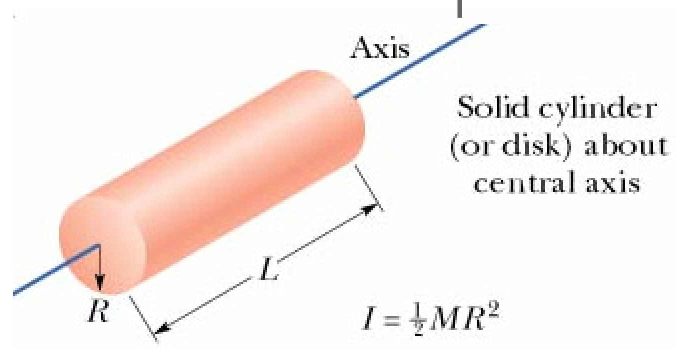
- **Circular Disk**

$$I_z = \int r_\perp^2 dm = \int_0^R \int_0^{2\pi} r^2 \overbrace{\underbrace{\left(\frac{M}{\pi R^2}\right)}_{\sigma} \underbrace{(r d\theta) dr}_{\text{Polar Coordinates}}}^{dm}$$

$$= \frac{2M}{R^2} \int_0^R r^3 dr = \frac{2M}{R^2} \left.\frac{r^4}{4}\right|_0^R = \frac{1}{2} M R^2$$

Note that the *mass element*, ***dm*** is 2D…





Why doesn't the third dimension matter here?

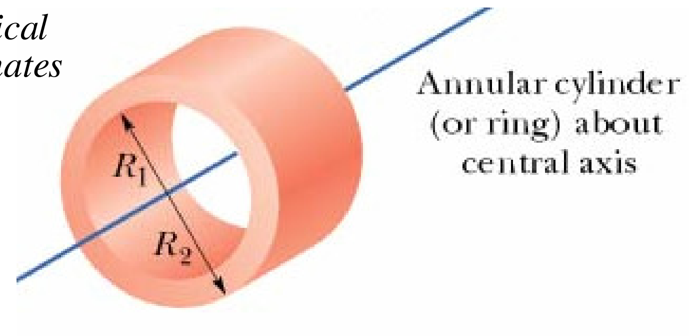
Calculating Moments - 3D

- **Massive Ring**

Note that the *mass element*, $\mathbf{dm}$ is 3D…

$$I_z = \int r_\perp^2 dm = \int_{R_1}^{R_2}\int_0^{2\pi}\int_0^L r^2 \underbrace{\left(\frac{M}{\pi\left(R_2^2 - R_1^2\right)L}\right)}_{\rho} \overbrace{\underbrace{(rd\theta)drdz}_{\text{Cylindrical Coordinates}}}^{dm}$$

$$I_{RING} = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$

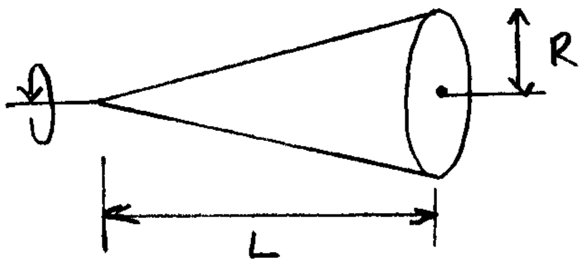


$$= \frac{2M}{\left(R_2^2 - R_1^2\right)}\int_{R_1}^{R_2} r^3 dr = \frac{2M}{\left(R_2^2 - R_1^2\right)}\left.\frac{r^4}{4}\right|_{R_1}^{R_2}$$

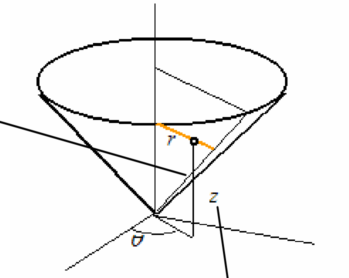
$$= \frac{1}{2}M\frac{R_2^4 - R_1^4}{\left(R_2^2 - R_1^2\right)} = \frac{1}{2}M\frac{\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)}{\left(R_2^2 - R_1^2\right)} = \frac{1}{2}M\left(R_2^2 + R_1^2\right)$$

Example - Inertia of a Cone

- **Uniform mass density**
  - Start with definition
  - Need volume of cone



$$I_z = \int r_\perp^2 dm = \rho \int r^2 dV = \frac{M}{\frac{1}{3}\pi R^2 L} \int_0^L dz \int_0^{2\pi} d\varphi \int_0^{\frac{R}{L}z} dr r^3$$



$$\Rightarrow \frac{6M}{R^2 L} \int_0^L dz \left(\frac{r^4}{4} \right) \Bigg|_0^{\frac{R}{L}z} = \frac{3M}{2R^2 L} \int_0^L dz \frac{R^4}{L^4} z^4 = \frac{3}{10} MR^2$$

Equation of outer surface is a line…

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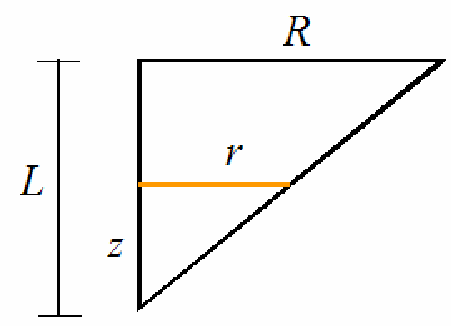
Finding the Volume of a Cone

- **Uniform mass density**
  - Start with definition

$$V = \int dV = \int_0^L dz \int_0^{2\pi} d\varphi \int_0^{\frac{R}{L}z} dr\, r = 2\pi \int_0^L dz \left. \frac{r^2}{2} \right|_0^{\frac{R}{L}z}$$

$$= \pi \frac{R^2}{L^2} \int_0^L z^2 dz = \pi \frac{R^2}{L^2} \left. \frac{z^3}{3} \right|_0^L = \frac{1}{3} \boxed{\pi R^2 L}$$

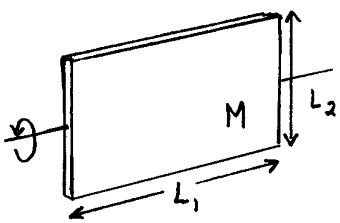


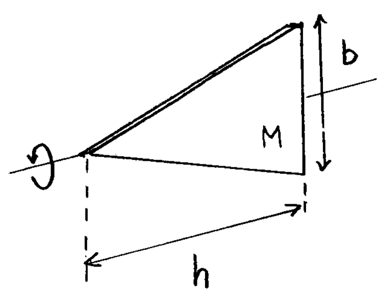
$$\frac{z}{L} = \frac{r}{R}$$

Use similar triangles to get equation of surface…

Activity – 2D/3D Integration

- **Use definition**
  - $$I = \int r_{\perp}^{2}\, dm$$
    - *Assume thickness is negligible…*
  - No tables!





Your Name (**Print**): ____________________ Date: ________
Group Members: ____________________ Group: ________

Rotational Inertia – Integration
(2D and 3D)

The rotational inertia (or moment of inertia) for a continuous mass distribution is defined as

$$I = \int_{\substack{\text{rigid}\\ \text{body}}} dI = \int_{\substack{\text{rigid}\\ \text{body}}} r^2\, dm$$

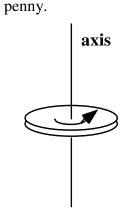
where r is the perpendicular distance of the mass dm from the axis of rotation.

The method to use is:

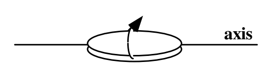
- draw a diagram showing the object and the axis of rotation
- choose a coordinate system that you think will be best to use
- pick a "slice" (or piece), dm, a perpendicular distance r from the axis of rotation; the piece should not be at the middle or the end of the object; show the distance r on your diagram
- write an expression for dm in terms of a small element of whatever coordinate(s) you have chosen
  - $dm = \sigma\, dA$ for 2-dimensional objects where σ is the mass/area and dA is the area of dm; dA will have to be expressed in terms of your spatial coordinates, for example, in Cartesian coordinates $dA = dx \cdot dy$)
  - $dm = \rho\, dV$ for 3D objects where ρ is the mass/volume and dV is the volume of dm; dV will have to be expressed in terms of your spatial coordinates, for example, in Cartesian coordinates $dV = dx \cdot dy \cdot dz$)
- substitute for dI in the integral definition
- do the integral.

1. Calculate I_{com} for a penny. Treat the penny as a flat, circular disk of mass M and radius R with a uniform mass distribution.
   a) Choose the axis to be perpendicular to the plane of the penny.
   b) Choose the axis to be in the plane of the penny.
   c) For both cases, also calculate the rotational inertia about a parallel axis passing through the edge of the penny.

(a)



(b)



HINT: Cut the disk up into small segments for which all of the mass is at the same distance from the axis. Then decide over what variable you have to integrate.

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