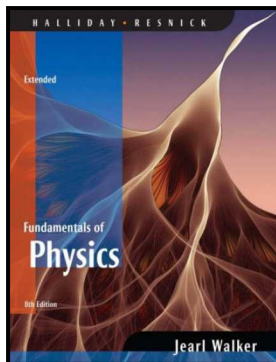


Workshop Physics

1017 - 312

University Physics II



Week 2 : Day 2

Outline

- ❑ **The Atwood Machine**
 - Rotational Equilibrium
 - Pre-lab Exercise
- ❑ **Rotational Inertia Laboratory**
 - Direct Calculation of Inertia
 - Experimental Determination of Inertia
- ❑ **Activity – Rotational Inertia**
 - Use Logger Pro to measure α
 - Measuring the applied torque
 - Determining the uncertainty
- ❑ **Laboratory Write-Up Guidelines**

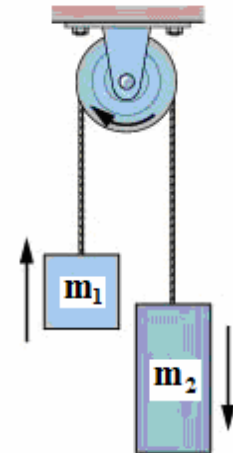
The Atwood Machine

□ The Atwood Machine

- Invented in 1784 by Reverend George Atwood as a laboratory experiment to verify the mechanical laws of uniformly accelerated motion.
(See [Wikipedia entry](#) for more...)

$$\sum \vec{F}, \sum \vec{\tau}$$

Free Body Diagram (FBD)



$$+T_1 - m_1g = +m_1a$$

$$r(+T_1 - T_2) = I(-\alpha)$$

$$+T_2 - m_2g = -m_2a$$

Atwood Solutions and Simulations

□ Solve the equations as follows:

$$(1) \quad +T_1 - m_1g = +m_1a \Rightarrow T_1 = m_1(g + a)$$

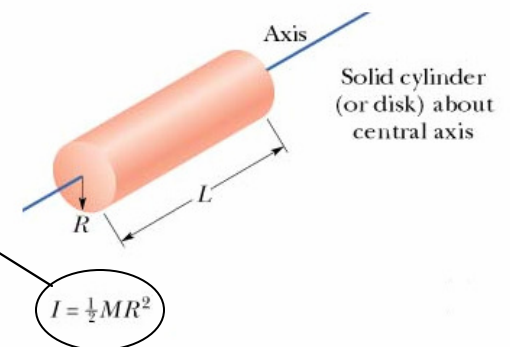
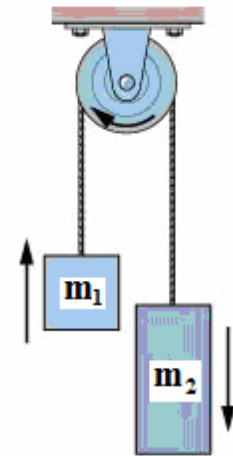
$$(2) \quad +T_2 - m_2g = -m_2a \Rightarrow T_2 = m_2(g - a)$$

$$(3) \quad r(+T_1 - T_2) = I(-\alpha)$$

$$\Rightarrow m_1(g + a) - m_2(g - a) = -\frac{I}{r} \left(\frac{a}{r} \right)$$

$$\Rightarrow (m_1 - m_2)g + (m_1 + m_2)a = -\frac{\frac{1}{2}Mr^2}{r^2} a$$

$$\Rightarrow a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{1}{2}M}$$

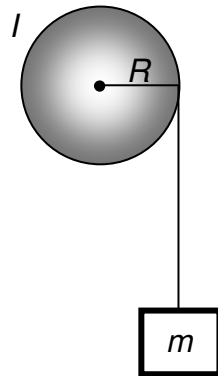


[Click here for Atwood Physlet](#)

Activity – Rotational Inertia

□ Pre-lab Exercise

- Set up equations of motion for mass-pulley system



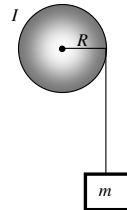
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Group Members: _____ Group: _____

Rotational Inertia - PreLab Exercise

This problem is similar to a configuration arising in an upcoming lab, so it's very important to go through this in detail and get the signs correct here.

Mass, String and Wheel

A wheel of radius R with moment of inertia I is mounted on a frictionless horizontal axle. You attach a light string to the wheel and wrap the string around the wheel. On the free end of the string there is a block of mass m attached. While you were wrapping the string, you held the wheel so it would not turn. At time $t = 0$ you release the wheel so the block begins falling, causing the wheel to turn.



- Lay down a coordinate system on the side of the diagram, and stick to it in all of the following steps.
- Draw the force diagram for the descending mass m in the space below. Let F_T represent the tension in the string.
- Write down Newton's Second Law for the block in terms of F_T , m , g , and its linear acceleration a .
- Draw the force diagram for the rotating wheel.
- Write down Newton's Second Law in rotational form for the wheel in terms of F_T , I , R , and its angular acceleration α . Leave I as an unknown, because we want to leave it completely general here and not assume the wheel is a thin disk (in fact, in the upcoming lab, it's not).

Pre-Lab Activity Results

□ Determine the moment of inertia

- Apply Newton's law in rotational form:

$$\sum \vec{\tau} = I\vec{\alpha} \Rightarrow -RF_T \sin(90^\circ) = I(-\alpha)$$

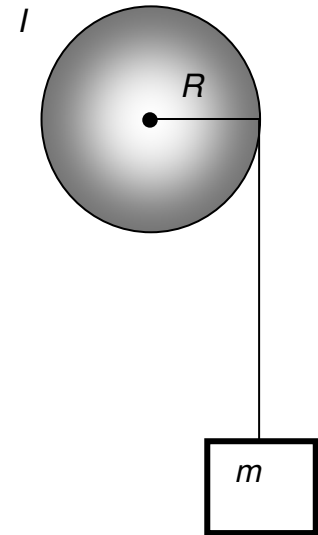
- Apply Newton's law in linear form:

$$\sum \vec{F} = m\vec{a} \Rightarrow F_T - mg = -ma$$

- Equate the linear and tangential acceleration

$$a_t = \alpha R$$

$$I_{\text{Object}} = \frac{mR(g - \alpha R)}{\alpha}$$



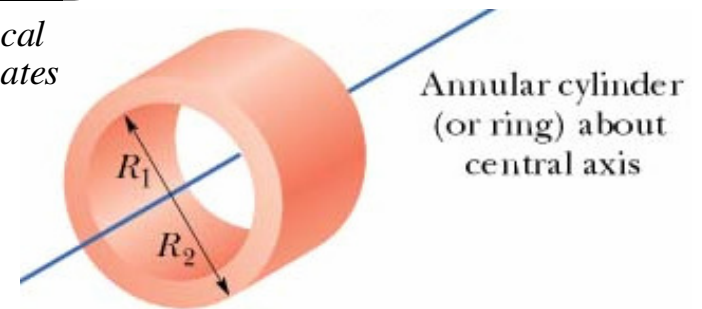
Calculating Moments - 3D

□ Massive Ring

Note that the *mass element, dm* is 3D...

$$I_z = \int r_{\perp}^2 dm = \int_0^R \int_0^{2\pi} \int_0^L r^2 \underbrace{\left(\frac{M}{\pi(R_2^2 - R_1^2)L} \right)}_{\rho} \underbrace{(rd\theta)drdz}_{\text{Cylindrical Coordinates}} dm$$

$$I_{RING} = \frac{1}{2} M (R_1^2 + R_2^2)$$



$$= \frac{2M}{(R_2^2 - R_1^2)} \int_0^R r^3 dr = \frac{2M}{(R_2^2 - R_1^2)} \frac{r^4}{4} \Big|_0^R$$

$$= \frac{1}{2} M \frac{R_2^4 - R_1^4}{(R_2^2 - R_1^2)} = \frac{1}{2} M \frac{(R_2^2 - R_1^2)(R_2^2 + R_1^2)}{(R_2^2 - R_1^2)} = \frac{1}{2} M (R_2^2 + R_1^2)$$

Experimental Techniques

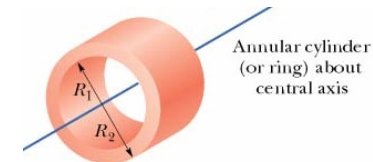
□ Find I experimentally for a compound object:

➤ Uniform disk & ring

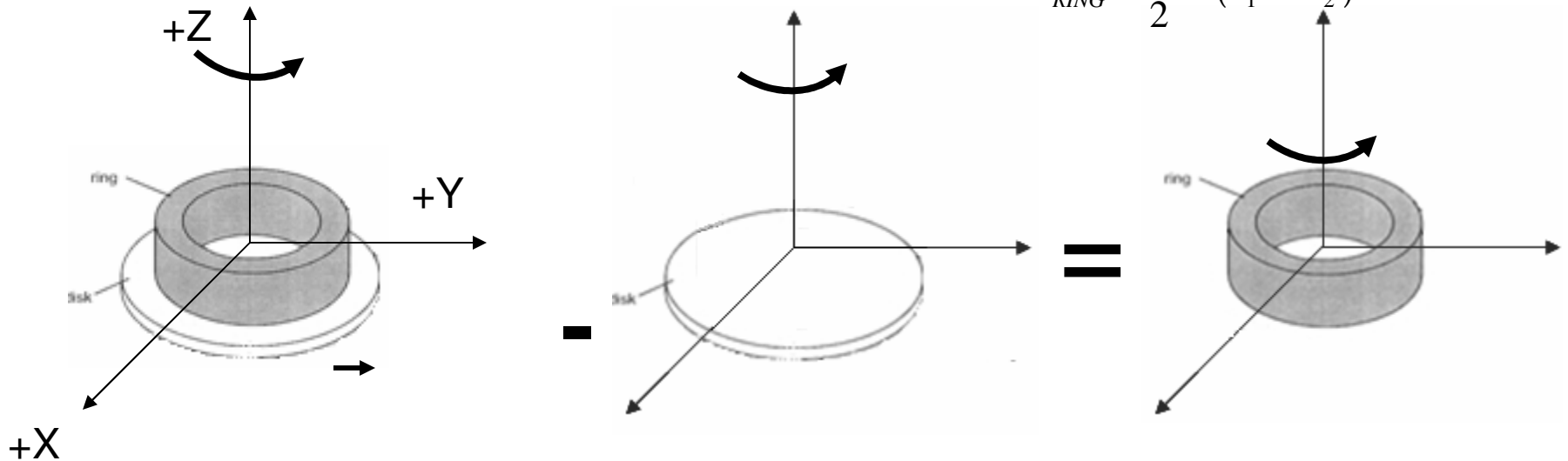
$$I_{TOT} = I_{DISK} + I_{RING}$$

$$\Rightarrow I_{RING} = I_{TOT} - I_{DISK}$$

Compare to Theoretical Value



$$I_{RING} = \frac{1}{2} M (R_1^2 + R_2^2)$$



Additional Measurements

□ Determining the *applied torque*:

- Measure radius of rotation
- Measure applied force

$$\vec{\tau} = \vec{r} \times \vec{F} = (r\hat{i}) \times (-mg\hat{k}) = rmg(+\hat{j})$$

□ Determining the *inertia*:

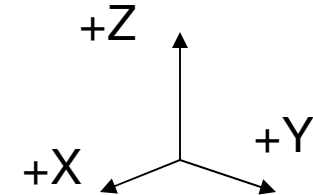
- Torque same for hanger + mass
- α varies for disk vs. disk + ring

$$I_{RING} = \frac{\tau_{TOT}}{\alpha_{TOT}} - \frac{\tau_{DISK}}{\alpha_{DISK}}$$

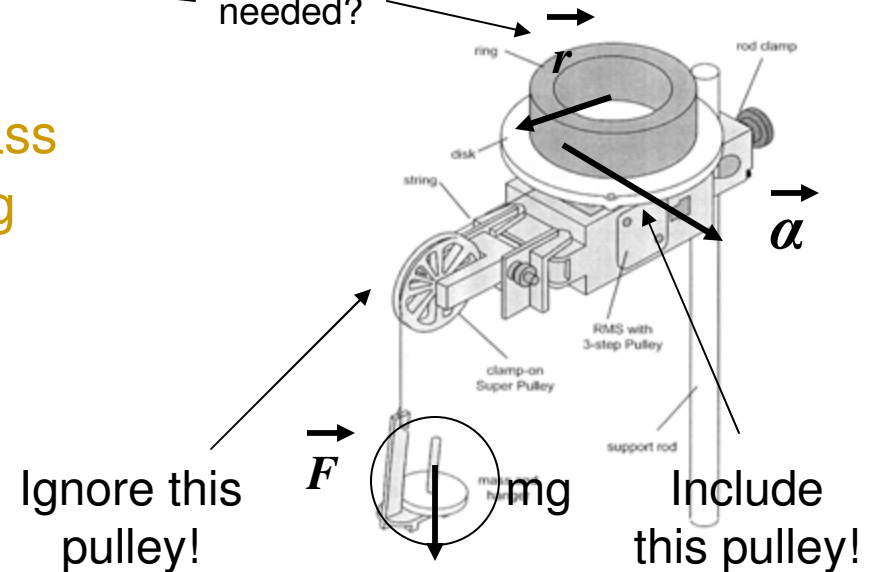
FOR EQUAL
MASSES

$$\Rightarrow I_{RING} = \frac{rmg(\alpha_{DISK} - \alpha_{TOT})}{\alpha_{TOT}\alpha_{DISK}}$$

Note: choose a coordinate system and stick to it...



What radius is needed?



Activity - Rotational Inertia

- ❑ Follow the Directions!
 - Complete steps 1 – 10 only
- ❑ Use Logger Pro to get the angular acceleration
 - Estimate uncertainty
- ❑ Use applied weight to calculate torque
 - Estimate uncertainty
- ❑ Determine inertia of ring and compare to expected value

Your Name (Print): _____ Date: _____
 Group Members: _____ Group: _____

Rotational Inertia

The goal here is an **accurate** determination of the moment of inertia of an annular ring and the moment of inertia of the two brass masses flush at the end of a rod (including the attachment screws, but **not** including the rod). This will be done by checking it two ways: (1) direct dynamical measurement (experimental) and (2) using the moment tables in the text (theoretical or static). **Both of these will have an associated uncertainty**, and there will be some overlap if they are in agreement.

The Rotary Motion Sensor (RMS) has an axle with a triple pulley. A string is wrapped around the RMS 3-step pulley, passed over a “massless” super pulley and attached to a hanging mass that will cause the system to accelerate. You will start this system from rest and measure the angular acceleration of the axle. From your measurements you should be able to do two things: (1) determine the rotational inertia of the RMS pulley/axle and of the object attached to it from these dynamical measurements; and (2) compare this result to the rotational inertia calculated theoretically from the mass and dimensions of the object.

Equipment list

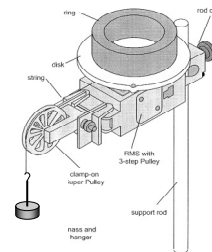


Figure 1. RMS with base plate and ring.

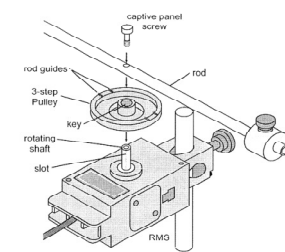


Figure 2. RMS with rod.

Rotary Motion Sensor with aluminum base plate, black ring, rod and masses for the rod.
 Team Physics Lab Computer with LoggerPro software and LabPro interface
 Super Pulley (which is so light we assume it's massless).
 Hanging Mass (use a **brass** 50 gram mass hanger, the plastic hangers break)
 Support rod and table clamp
 Physics thread (which is so light we assume it's massless).

Do not lose the screws! This happens in every lab. If you don't remove them from the rod, then you won't lose them.

Using Logger Pro

□ Use Logger Pro software

- The Rotary Motion Sensor (RMS) has an axle with a triple pulley
- Record the angular displacement and angular velocity as a function of time – $\theta(t)$, $\omega(t)$

- *Software located in share drive: (See page 2...)*

- Go to My Computer... Student Shares... University Physics Students... Team Physics 312... LoggerPro. In that folder you will find a file named: "Moment_of_Inertia"

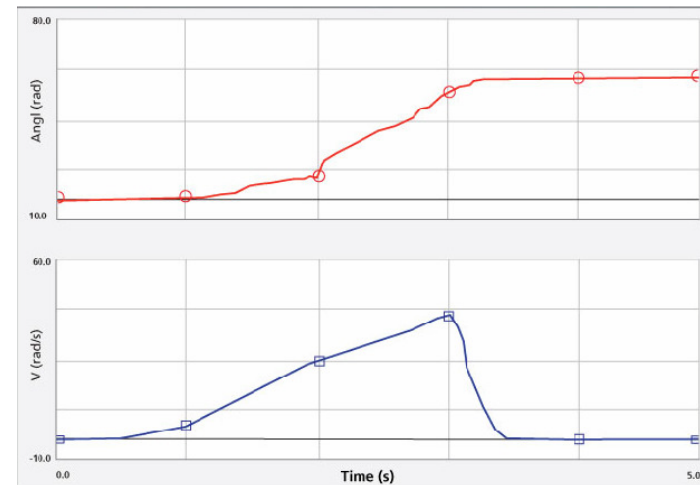
- *Use the appropriate curve fit*

- Angular displacement equation:

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

- Angular velocity equation:

$$\omega(t) = \omega_0 + \alpha t$$



Summarizing Your Results

- Find the *experimental uncertainty* for the inertia of the ring.
 - Use actual values of ring:
 - Measure mass of ring
 - Measure inner radius
 - Measure outer radius

Propagation of Error

$$I_{RING} = \frac{1}{2} M (R_1^2 + R_2^2)$$
$$\Rightarrow \Delta I_{RING} = ?$$

- Assess the validity of the experiment
 - Write experimental value attained:

$$I_{Experimental} = I_{RING} \pm \Delta I_{RING}$$

- Determine if percent error is accounted for:

$$\% Error = \frac{|I_{Theoretical} - I_{Experimental}|}{I_{Theoretical} + I_{Experimental}} \times 100$$

Writing up your results

□ Team write-ups (Guidelines Available Online)

- The lab write-ups will be graded on a ten point basis:

| | |
|-------------------------|---|
| Abstract: | 2 marks (major) |
| Theory: | 1 mark |
| Data tables: | 1 mark (record uncertainties and units) |
| Uncertainties: | 2 marks (for experimental & theoretical values) |
| Sample calculations: | 1 mark (show one example) |
| Graphs/LoggerPro plots: | 1 mark (labels and units) |
| Discussion of results: | 1 mark |
| Neatness and Clarity: | 1 mark |
| <i>Total</i> | 10 marks |

- All reports due one week from date of activity performance
- Group members must be present to receive credit