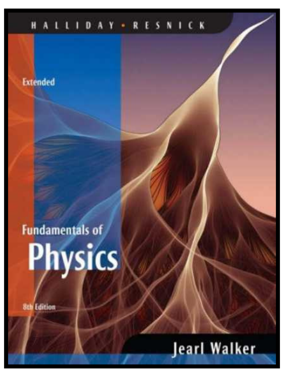
Workshop Physics 1017 - 312

University Physics II



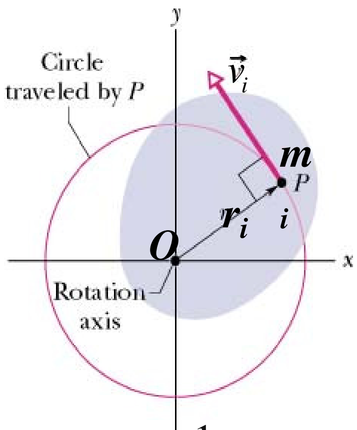
Week 2 : Day 3

RIT

Outline

- **The Energy of Rotation**
  - Power from Rotation
  - Work and Torque
  - Power Generation and Use
    - *From helicopters to wind turbines…*
- **Rolling Motion**
  - COM Rotation and Translation
  - Review – Sliding Blocks & Friction
  - The Role of Friction in Rolling
- **Conservation of Energy**
  - Review - Kinetic and potential energy
  - Activity – Rolling and Ramps
  - Applying what we learned…

Energy of Rotation



Consider the rotating rigid body shown in the figure.

We divide the body into parts of masses $m_1, m_2, m_3, \ldots, m_i, \ldots.$

The part (or "element") at P has an index i and mass m_i.

The kinetic energy of rotation is the sum of the kinetic

energies of the parts: $K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \ldots.$

$K = \sum_i \frac{1}{2} m_i v_i^2$ The speed of the ith element $v_i = \omega r_i \rightarrow K = \sum_i \frac{1}{2} m_i \left(\omega r_i\right)^2.$

$K = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$ The term $I = \sum_i m_i r_i^2$ is known as

rotational inertia or moment of inertia about the axis of rotation.

$$\Rightarrow \quad K = \frac{1}{2} I \omega^2$$

Power from Rotation

- **Power is defined as the amount of work done in a given time. The *instantaneous power* is:**

$$P = {dW}/{dt}$$

- **For pure rotational motion the work is equal to the kinetic energy of rotation:**

$$W = K = \frac{1}{2} I\omega^2$$

- **The time rate of change of the work can be found directly as follows:**

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = \frac{1}{2} I \frac{d}{dt}\omega^2 = \frac{1}{2} I \left(2\omega \underbrace{\frac{d\omega}{dt}}_{\alpha}\right)$$

$$\Rightarrow P = I\omega\alpha = \overbrace{\tau}^{I\alpha}\omega$$

units give form…

The *units* of **Power**:

$$[P] \equiv Watt = \frac{J}{s}$$

$$= \frac{N \cdot m}{s}$$

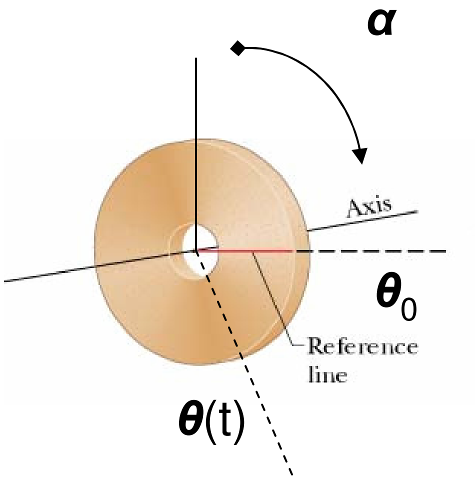
Work and Torque

- Recall that the units of torque and work are the same
  - Thus means that there is likely a relationship between them…
- Consider the power generated by a rotating grinding wheel:

$$\Rightarrow P \equiv \frac{dW}{dt} = \tau\omega$$

$$\Rightarrow dW = \tau\omega dt = \tau d\theta$$

$$\Rightarrow \int_{W_i}^{W_f} dW = \tau \int_{\theta_i}^{\theta_f} d\theta \;\} \Rightarrow \boxed{\Delta W = \tau \Delta\theta}$$

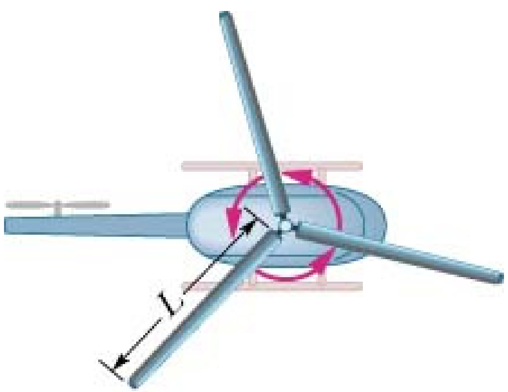


$$\tau = I\alpha = \underbrace{\left(\tfrac{1}{2}Mr^2\right)}_{Table}\alpha$$

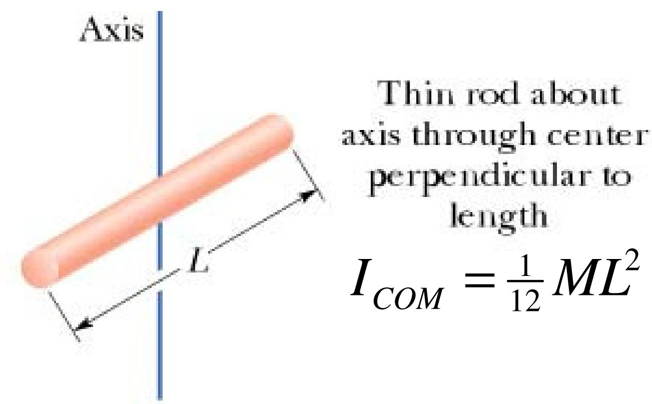
Power Generation and Use

- **The ENERGY of rotating objects can be utilized to generate/use POWER**
  - Need moment of inertia to calculate power:

$$P = \tau\omega = (I\alpha)\omega = 3\left(\frac{1}{3}ML^2\right)\left(\frac{\omega}{T}\right)\omega = \frac{M}{T}L^2\omega^2$$



From
HELICOPTERS…

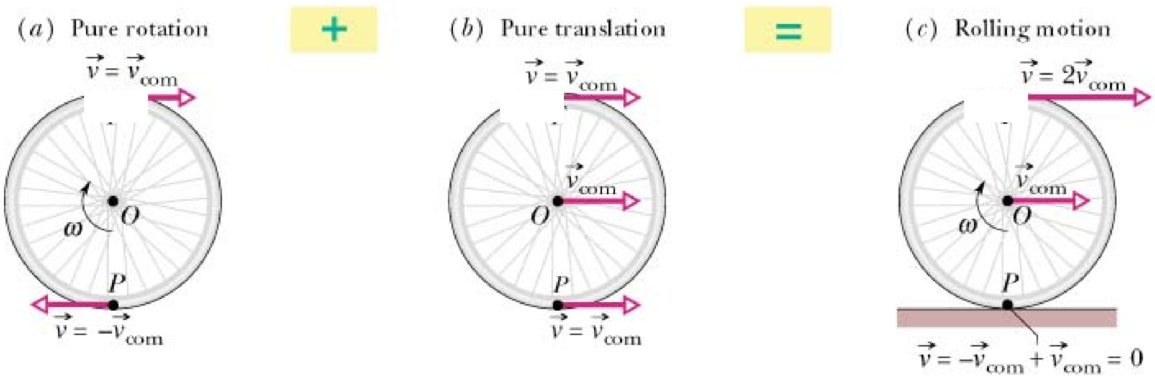


$$I_{Blade} = I_{COM} + ML^2 = \tfrac{1}{3}ML^2$$

To
WIND TURBINES…

Rolling Energy

- **Rolling objects**
  - When an object rolls it translates and rotates



  - Therefore it has two forms of kinetic energy:
    - *Translational:* $K_{Trans} = \frac{1}{2} m v_{com}^2$
    - *Rotational:* $K_{Rot} = \frac{1}{2} I w^2$

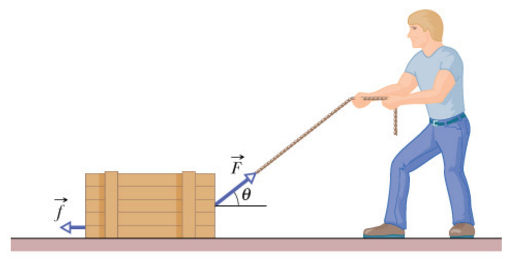
$$\Rightarrow K = K_{Trans} + K_{Rot}$$

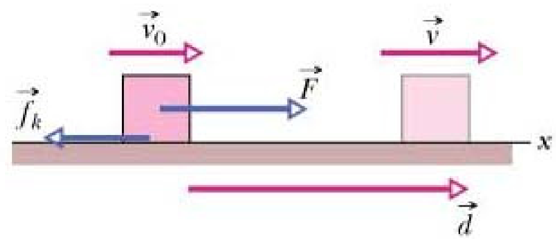
Review – Sliding Blocks & Friction

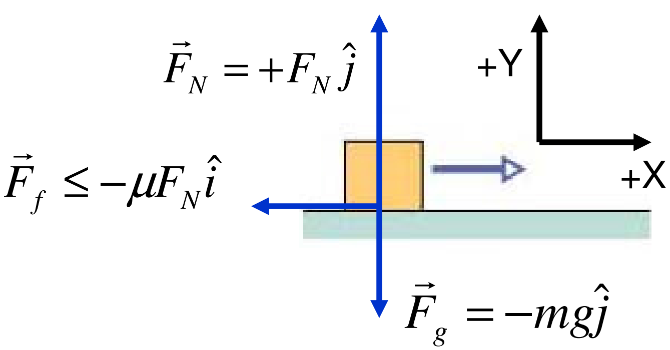
- **Friction forces**
  - Static coefficient
  - Kinetic coefficient

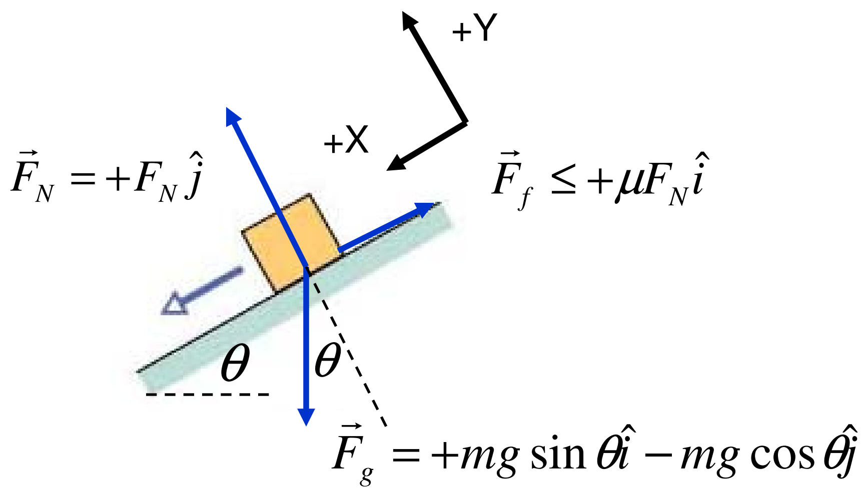
$$\Rightarrow \vec{F}_f \leq \mu \vec{F}_N$$

- **Free body diagrams**
  - Coordinate choice very important!









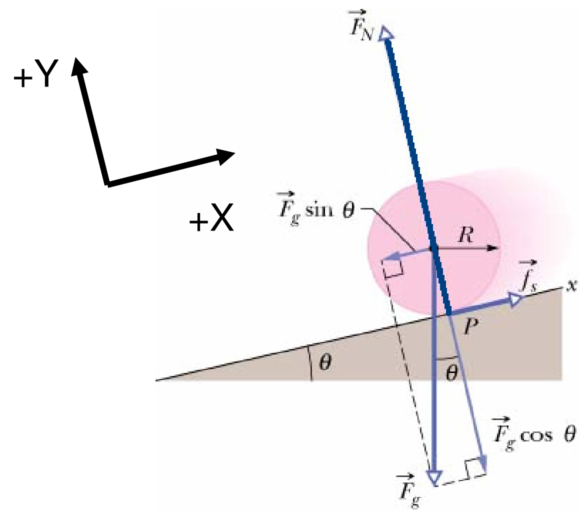
The Role of Friction in Rolling

- Frictional forces for rolling
  - Act at a single point of contact
  - Sole source of rotation
    - *w/o friction it would "slip or slide"*
- Problem set-up
  - Need Newton's second law
    - *Linear form*

$$\sum F_x = F_s - Mg\sin\theta = Ma_{com,x}$$

    - *Rotational form:*

$$\sum \tau = RF_s = I_{com}\alpha$$



Finding the Solution…

- **To solve for acceleration**
  - Link the variables

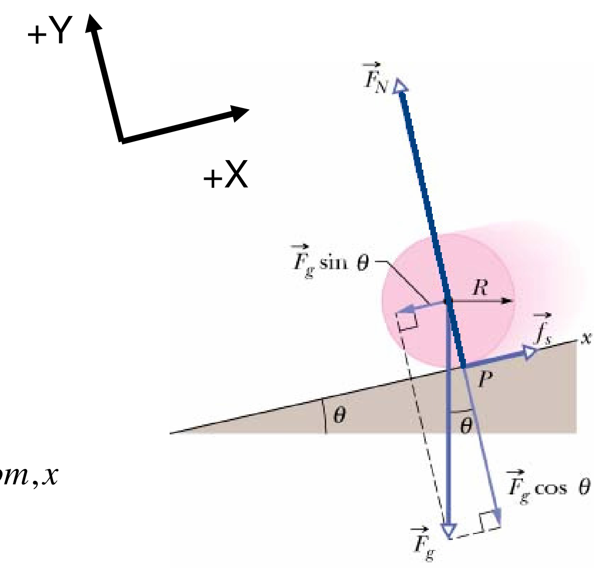
$$a_{com,x} \equiv \alpha R$$

  - And substitute

$$F_s - Mg\sin\theta = Ma_{com,x}$$

$$\Rightarrow -I_{com}\frac{a_{com,x}}{R^2} - Mg\sin\theta = Ma_{com,x}$$

$$\Rightarrow a_{com,x} = -\frac{g\sin\theta}{1+\dfrac{I_{com}}{MR^2}}$$



Conservation of Energy

- **Potential energy**
  - Gravitational:

$$\Delta U = mg\Delta h = mg\left(h_f - h_i\right)$$

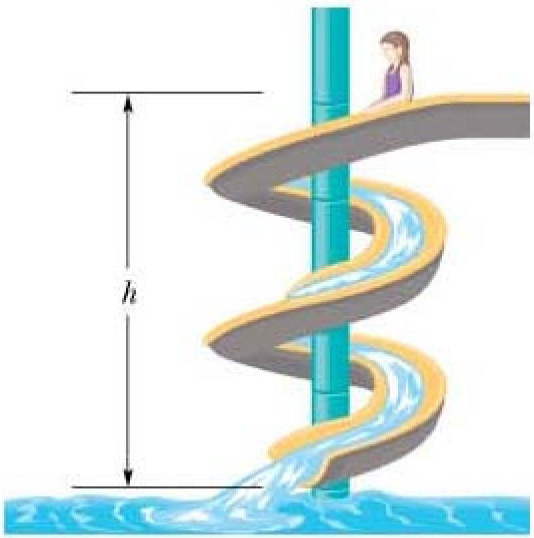
- **Conservation Principle**
  - Final equals initial if n loss

$$\Delta K + \Delta U = 0$$
$$\left(K_f - K_i\right) + \left(U_f - U_i\right) = 0$$
$$\Rightarrow K_f + U_f = K_i + U_i$$

  - Solve for unknown quantity

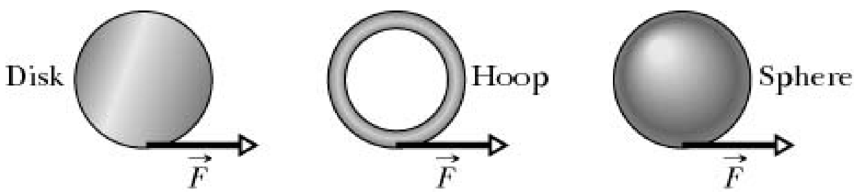
$$\frac{1}{2}m(v)^2 + mg(0) = \frac{1}{2}m(0)^2 + mg(h)$$
$$\Rightarrow v = \sqrt{2gh}$$

How fast is girl traveling at bottom?



Activity – Rolling and Ramps

- **Place these objects on ramp**
  - Release from rest at same height
  - Which one wins



$$I_{Disk} = \frac{1}{2}MR^2 \qquad I_{Hoop} = MR^2 \qquad I_{Sphere} = \frac{2}{5}MR^2$$

$$\Rightarrow I_{Object} = k_{Object}MR^2$$

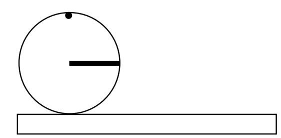
- **Predict the WINNER!**
  - Use energy principles

$$K = K_{Trans} + K_{Rot} \Rightarrow \Delta K + \Delta U = 0$$

Your Name (**Print**): ______________________ Date: ________
Group Members: ______________________
______________________ Group: ________

Rolling and Ramps

Warm-up: Consider a rolling wheel. What is the instantaneous linear velocity of a point at the bottom of the wheel in contact with the ground?



What is the relation between the velocity of the wheel as a whole (essentially the linear velocity of the axle) and the wheel's angular velocity?

What is the instantaneous linear velocity of a point at the top of the wheel, relative to the ground, in terms of the angular velocity?

A hollow 45.8 g cylinder of radius R = 2.25 cm rolls down a 27.3º ramp starting from a height of h = 1.50 m. The cylinder does not slip or slide as it rolls down. What is the linear speed v of the cylinder at the bottom of the ramp?

Predicting the winner…

- **Apply energy conservation**
  - Find the final velocity after fall

$$K_i + U_i = K_f + U_f$$

Starts from rest → (0); Final height is "zero" → (0)

$$(0) + (Mgh) = \left(\frac{1}{2}Mv^2 + \frac{1}{2}I_{Object}\omega^2\right) + (0)$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{M}{M}\right)I_{Object}\left(\frac{v}{R}\right)^2$$

Connect angular and linear variables…

$$2gh = v^2 + \frac{I_{Object}}{MR^2}v^2$$

$$\Rightarrow v = \sqrt{\frac{2gh}{1 + I_{Object}/MR^2}}$$

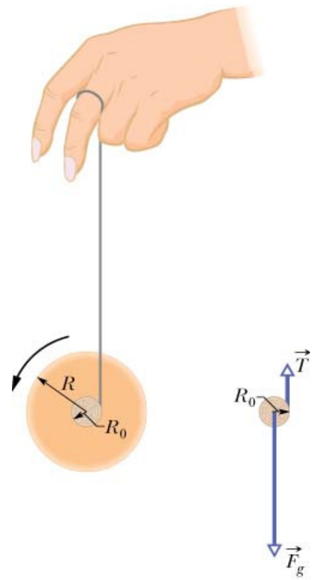
*Object with greatest inertia loses the race!

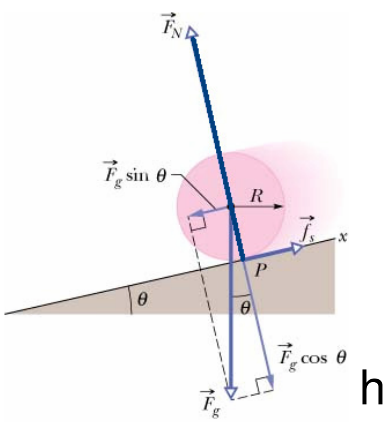
Applying what we learned…

- **There are many applications**
  - The YO-YO
    - *Same solution w/o ramp*

$$\Rightarrow a_{com} \equiv -\frac{g\sin 90^{\circ}}{1+I_{com}/MR^2}$$

  - Find velocity
    - *Use linear equations*





$$v^2 \equiv v_0^2 + 2a\Delta x$$

$$\Rightarrow v^2 = \frac{2g(\sin\theta\Delta x)}{1+I_{com}/MR^2} = \frac{2gh}{1+I_{com}/MR^2}$$

This is the same answer for $\boldsymbol{v}$ that we get using energy!