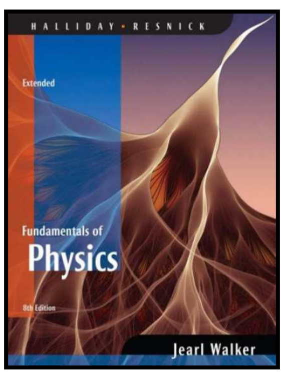
University Physics II



Week 3 : Day 1

Outline

- **Rotational Dynamics**
  - Angular variables: θ, ω, and α
- **Relation b/w linear and angular variables**
  - Tangential velocity
  - Tangential and centripetal acceleration
- **Moments of Inertia**
  - Vector directions
  - Calculating moments
  - Torque, work and power
- **Translational and rotational energy**
  - Conservation of energy

Rotational Variables

- **Angular variables include**
  - Angular displacement, θ
  - Angular velocity, ω
  - Angular acceleration, α

$$\omega \equiv \frac{d\theta}{dt}$$

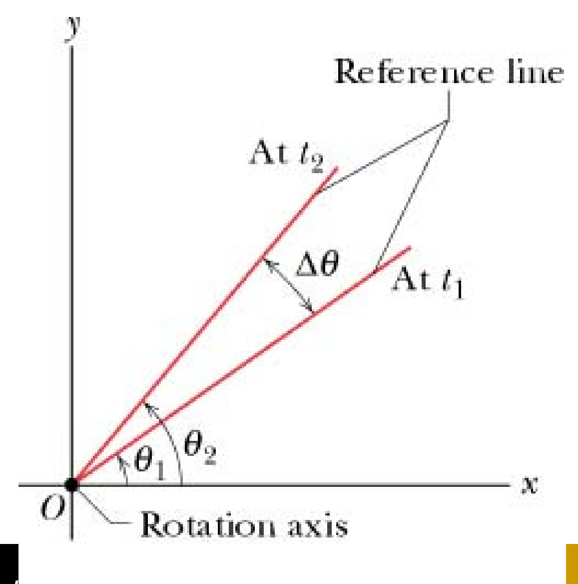
$$\alpha \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

"[var]" = units of var

$$[\theta] = rad$$

$$[\omega] = \frac{rad}{s}$$

$$[\alpha] = \frac{rad}{s^2}$$



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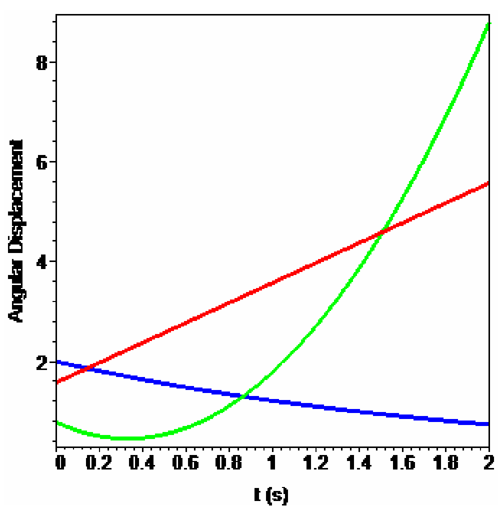
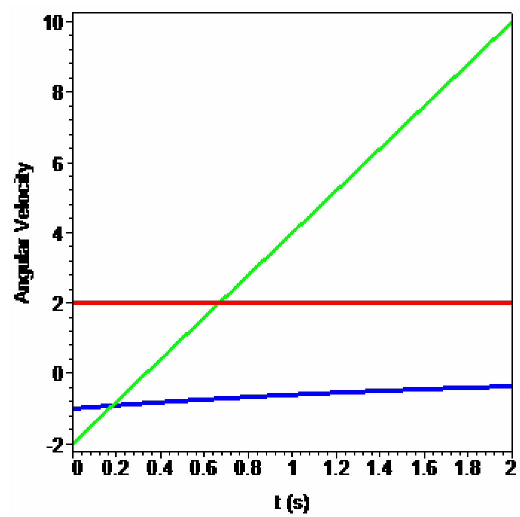
Angular Paths - Examples

- Consider three different angular paths:
  - **Path 1** $\theta(t) = 2t + \pi/2, \omega(t) = 2, \alpha(t) = 0$
  - **Path 2** $\theta(t) = 3t^2 - 2t + \pi/4, \omega(t) = 6t - 2, \alpha(t) = 6$
  - **Path 3** $\theta(t) = 2e^{-t/2}, \omega(t) = -e^{-t/2}, \alpha(t) = \frac{1}{2}e^{-t/2}$

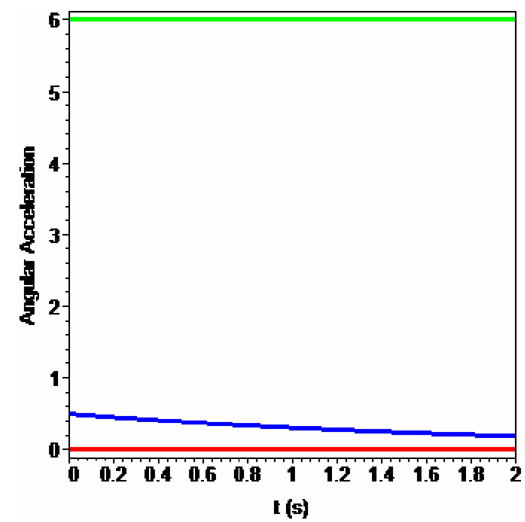
$\theta(t)$



$\omega(t)$



$\alpha(t)$



Angular Equations of Motion

- For constant acceleration:

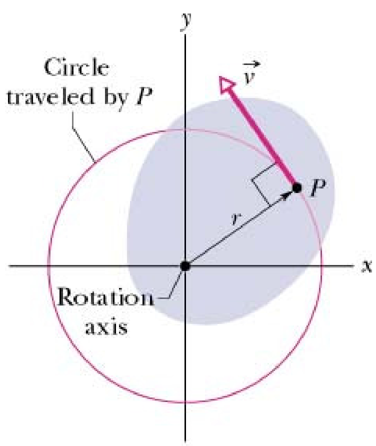
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Angular and Linear Relations

- **Rotational motion about fixed axis**
  - Circular motion for constant radius
  - Period of motion is also constant



velocity

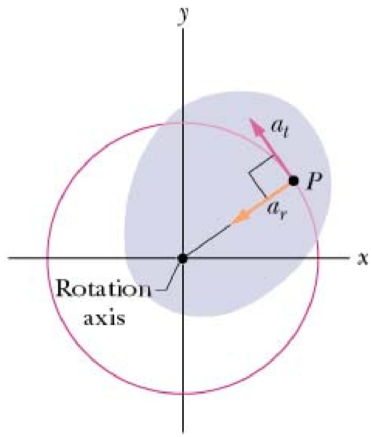
$$\frac{dS}{dt} = r\frac{d\theta}{dt} = r\omega$$

$$\Rightarrow v_t = r\omega$$

$$\Rightarrow v_t = \frac{2\pi r}{T} = \omega r$$

$$\Rightarrow a_r = \frac{v_t^2}{r} = r\omega^2$$

$$\Rightarrow \omega = \frac{2\pi}{T}$$



acceleration

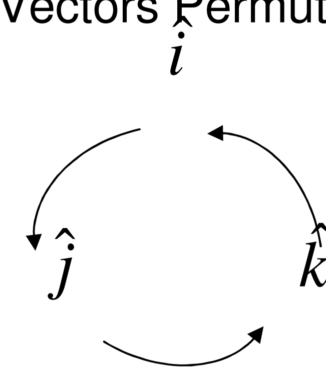
$$a_t = \frac{dv_t}{dt} = r\frac{d\omega}{dt} = r\alpha$$

$$\Rightarrow a_t = r\alpha$$

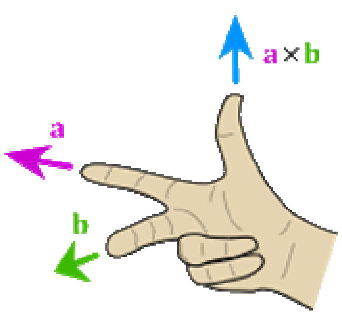
Vector Directions

- **To determine the direction of a rotation it is sometime helpful to introduce the *unit vector* notation for the different directions:**
  - $\hat{i}$ - Direction along X-Axis
  - $\hat{j}$ - Direction along Y-Axis
  - $\hat{k}$ - Direction along Z-Axis

Vectors Permute



Right-Hand Rule



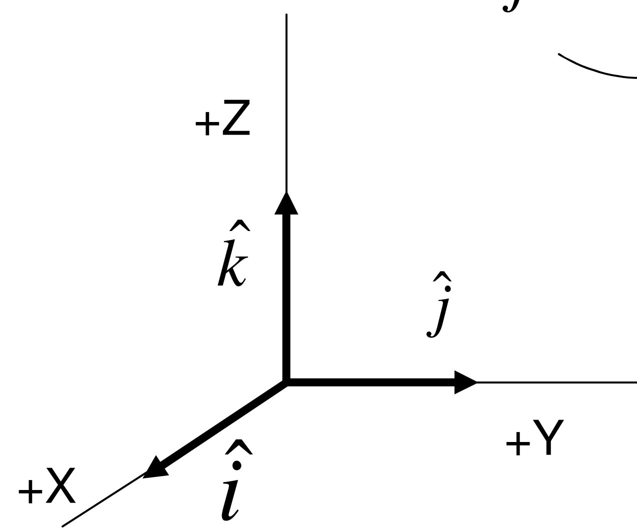


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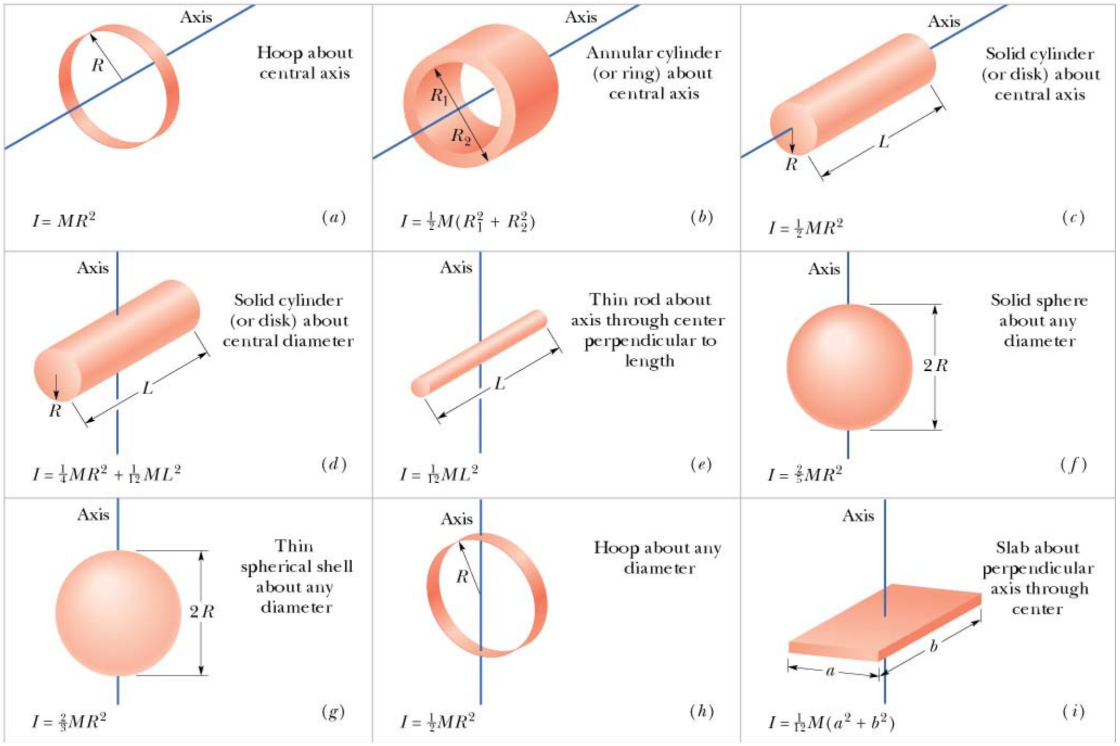
Table of Moments

- Moments through COM for common objects



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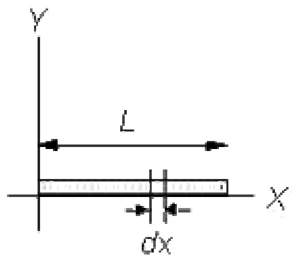
Calculating Moments - ID

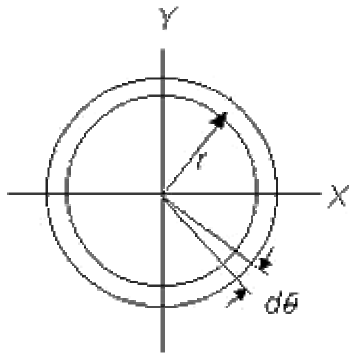
- Thin Rod

$$I_y = \int_0^L r_\perp^2 dm = \int_0^L x^2 \left(\underbrace{\frac{m}{L}}_{\lambda} \underbrace{dx}_{dl} \right) = \frac{m}{L} \frac{x^3}{3} \bigg|_0^L = \frac{1}{3} mL^2$$



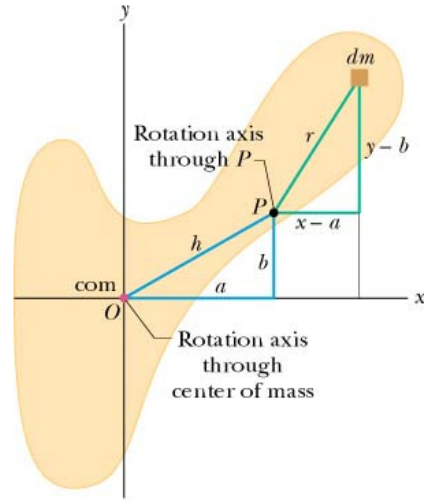
- Circular Ring

$$I_z = \int_0^{2\pi} r_\perp^2 dm = \int_0^{2\pi} r^2 \left(\underbrace{\frac{m}{2\pi r}}_{\lambda} \underbrace{r d\theta}_{dl} \right) = \frac{mr^2}{2\pi} \int_0^{2\pi} d\theta = mr^2$$

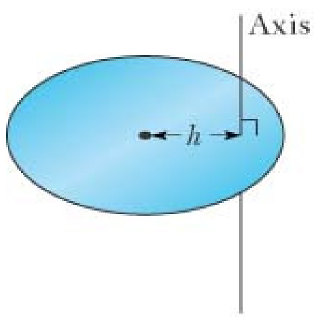


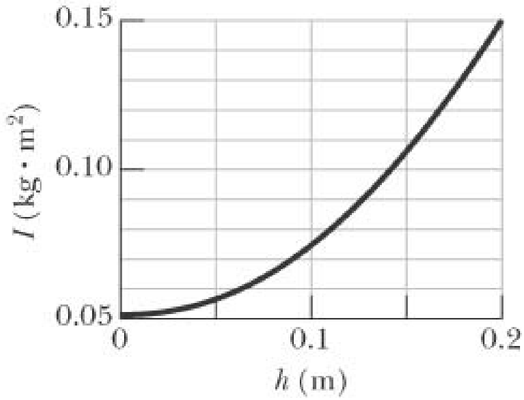
Parallel Axis Theorem

- **Inertia Calculations**
  - Moments are additive



$$I = I_{COM} + Mh^2$$





Power from Rotation

- **Power is defined as the amount of work done in a given time. The *instantaneous power* is:**

$$P = {dW}/{dt}$$

- **For pure rotational motion the work is equal to the kinetic energy of rotation:**

$$W = K = \frac{1}{2} I\omega^2$$

- **The time rate of change of the work can be found directly as follows:**

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = \frac{1}{2} I \frac{d}{dt}\omega^2 = \frac{1}{2} I\left(2\omega\underbrace{\frac{d\omega}{dt}}_{\alpha}\right)$$

$$\Rightarrow P = I\omega\alpha = \overbrace{\tau}^{I\alpha}\omega$$

The *units* of **Power**:

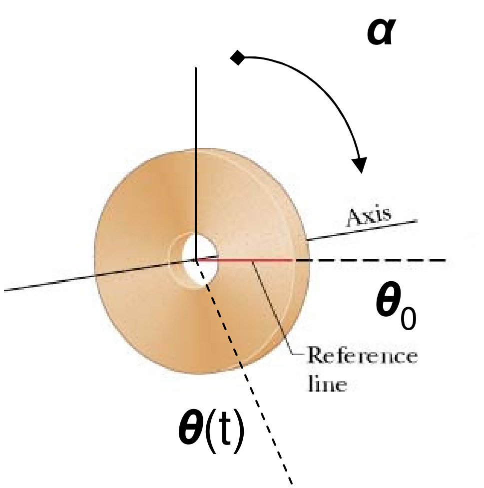
$$[P] \equiv Watt = \frac{J}{s}$$

$$= \frac{N \cdot m}{s}$$

units give form…

Work and Torque

- Recall that the units of torque and work are the same
  - Thus means that there is likely a relationship between them…
- Consider the power generated by a rotating grinding wheel:



$$\tau = I\alpha = \underbrace{\left(\tfrac{1}{2}Mr^2\right)}_{Table}\alpha$$

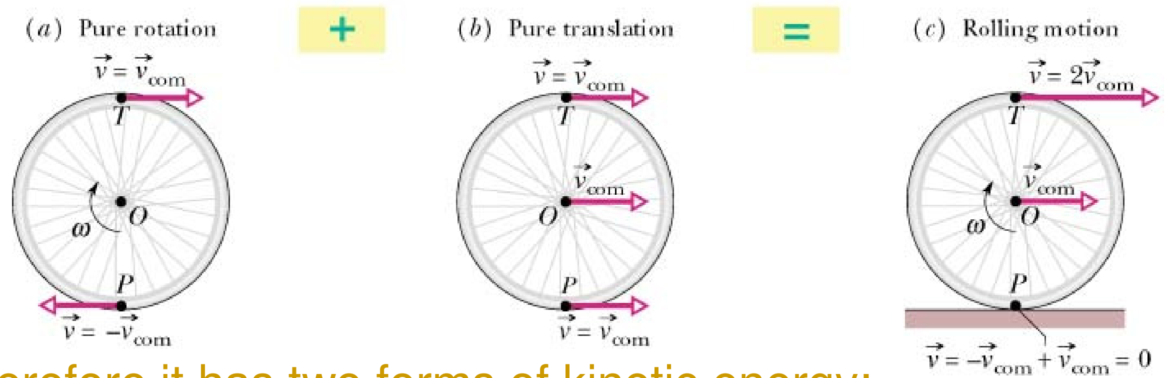
$$\Rightarrow P \equiv \frac{dW}{dt} = \tau\omega$$

$$\Rightarrow dW = \tau\omega dt = \tau d\theta$$

$$\Rightarrow \int_{W_i}^{W_f} dW = \tau \int_{\theta_i}^{\theta_f} d\theta \;\}\Rightarrow \boxed{\Delta W = \tau\Delta\theta}$$

Rolling Energy

- **Rolling objects**
  - When an object rolls it translates and rotates

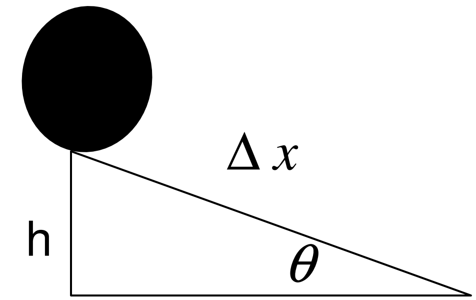


  - Therefore it has two forms of kinetic energy:
    - *Translational:* $K_{Trans} = \frac{1}{2} m v_{com}^2$
    - *Rotational:* $K_{Rot} = \frac{1}{2} I w^2$

$$\Rightarrow K = K_{Trans} + K_{Rot}$$

Conservation of Energy

- **Apply energy conservation**
  - Find the final velocity after fall



$$K_i + U_i = K_f + U_f$$

Starts from rest →

$$(0) + (Mgh) = \left(\frac{1}{2}Mv^2 + \frac{1}{2}I_{Object}\omega^2\right) + (0)$$

← Final height is "zero"

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{M}{M}\right)I_{Object}\left(\frac{v}{R}\right)^2$$

← Connect angular and linear variables…

$$2gh = v^2 + \frac{I_{Object}}{MR^2}v^2$$

$$\Rightarrow v = \sqrt{\frac{2gh}{1 + \frac{I_{Object}}{MR^2}}}$$