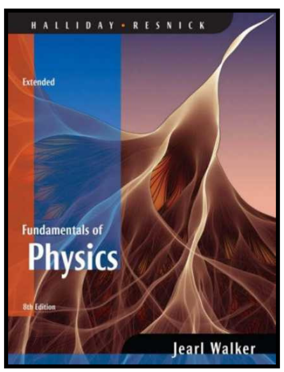
University Physics II

Week 4 : Day 2



Outline

- **Defining equilibrium**
  - Conditions needed for equilibrium
    - *Sum of forces*
    - *Sum of torques*
  - Unstable equilibrium
    - *COM & COG*
- **Problem Recipe**
  - *Static* equilibrium examples
- **Activity – Forces on Extended Bodies**
  - Examine forces and torques for body in equilibrium

Defining Equilibrium

We say that an object is in equilibrium when the following two conditions are satisfied:

1. The linear momentum $\vec{P}$ of the center of mass is constant.
2. The angular momentum $\vec{L}$ about the center of mass or any other point is a constant.

Our concern in this chapter is with situations in which $\vec{P} = 0$ and $\vec{L} = 0$.
That is, we are interested in objects that are not moving in any way (this includes translational as well as rotational motion) in the reference frame from which we observe them. Such objects are said to be in static equilibrium.
In Chapter 8 we differentiated between stable and unstable static equilibrium.
If a body that is in static equilibrium is displaced slightly from this position the forces on it may return it to its old position. In this case we say that the equilibrium is stable. If the body does not return to its old position then the equilibrium is unstable.

Conditions of Equilibrium

In Chapter 9 we calculated the rate of change for the linear momentum of the center of mass of an object, $\frac{d\vec{P}}{dt} = \vec{F}_{\text{net}}$. If an object is in translational equilibrium then

$\vec{P} =$ constant and thus $\frac{d\vec{P}}{dt} = 0 \rightarrow \quad \vec{F}_{\text{net}} = 0.$

In Chapter 11 we analyzed rotational motion and saw that Newton's second law takes the form $\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$. For an object in rotational equilibrium we have: $\vec{L} =$ constant

$\rightarrow \frac{d\vec{L}}{dt} = 0 \rightarrow \quad \vec{\tau}_{\text{net}} = 0.$

1. The vector sum of all the external forces on the body must be zero.
2. The vector sum of all the external torques that act on the body measured about any point must be zero.

Simplifying Assumptions

In component form the conditions of equilibrium are:

Balance of forces: $F_{\text{net},x} = 0 \quad F_{\text{net},y} = 0 \quad F_{\text{net},z} = 0$

Balance of torques: $\tau_{\text{net},x} = 0 \quad \tau_{\text{net},y} = 0 \quad \tau_{\text{net},z} = 0$

We shall simplify matters by considering only problems in which all the forces that act on the body lie in the *xy*-plane. This means that the only torques generated by these forces tend to cause rotation about an axis parallel to the *z*-axis. With this assumption the conditions for equilibrium become:

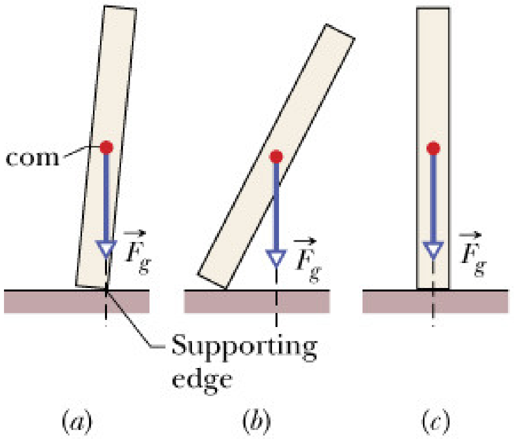
Balance of forces: $F_{\text{net},x} = 0 \quad F_{\text{net},y} = 0$

Balance of torques: $\tau_{\text{net},z} = 0$

Here $\tau_{\text{net},z}$ is the net torque produced by all external forces either about the *z*-axis or about any axis parallel to it.

Finally, for static equilibrium the linear momentum $\vec{P}$ of the center of mass must be zero: $\vec{P} = 0.$

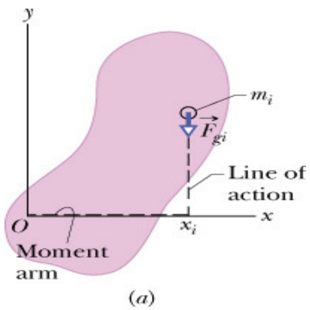
Unstable Equilibrium

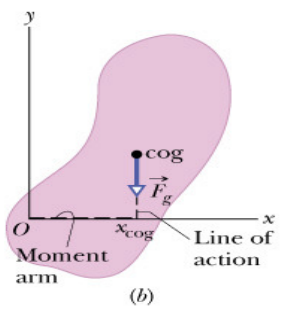


An example of unstable equilibrium is shown in the figures. In fig. a we balance a domino with the domino's center of mass vertically above the supporting edge. The torque of the gravitational force $\vec{F}_g$ about the supporting edge is zero because the line of action of $\vec{F}_g$ passes through the edge.

Thus the domino is in equilibrium. Even a slight force on the domino ends the equilibrium. As the line of action of $\vec{F}_g$ moves to one side of the supporting edge (see fig. b) the torque due to $\vec{F}_g$ is nonzero and the domino rotates in the clockwise direction away from its equilibrium position of fig. a.
The domino in fig. a is in a position of unstable equilibrium.
The domino is fig. c is not quite as unstable. To topple the domino the applied force would have to rotate it through and beyond the position of fig. a.

Center of Gravity





The gravitational force acting on an extended body is the vector sum of the gravitational forces acting on the individual elements of the body. The gravitational force $\vec{F}_g$ on a body effectively acts at a single point known as the center of gravity of the body. Here "effectively" has the following meaning: If the individual gravitational forces on the elements of the body are turned off and replaced by $\vec{F}_g$ acting at the center of gravity, then the net force and the net torque about any point on the body do not change. We shall prove that if the acceleration of gravity $\vec{g}$ is the same for all the elements of the body then the center of gravity coincides with the center of mass. This is a reasonable approximation for objects near the surface of the Earth because $\vec{g}$ changes very little.

If we set $g_i = g$ for all the elements $\rightarrow x_{cog} = \dfrac{\sum_i m_i x_i}{\sum_i m_i} = x_{com}$.

Problem Recipe

1. Draw a force diagram. (Label the axes.)
2. Choose a **convenient** origin O

   have one of the unknown force acting at *O*

3. Sign of the torque τ for each force:

   - Force induces clockwise (CW) rotation

   + Force induces counterclockwise (CCW) rotation

4. Apply equilibrium conditions:

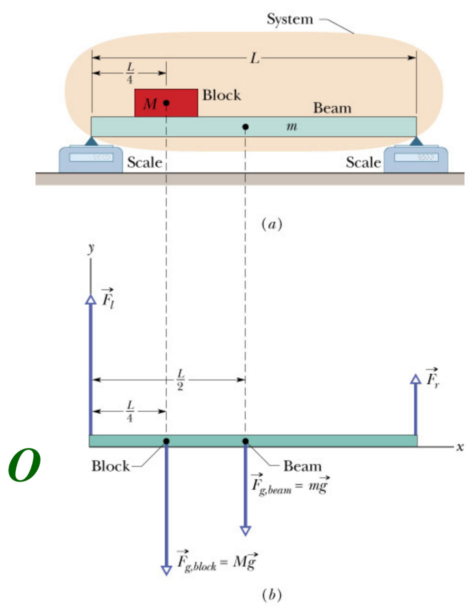
$$F_{\text{net},x} = 0 \qquad F_{\text{net},y} = 0$$

$$\tau_{\text{net},z} = 0$$

5. Make sure that

   number of unknowns = number of equations

Example – Beam Balance



Sample Problem 12-1. A uniform beam of length L and mass m = 1.8 kg is at rest on two scales.
A uniform block of mass M = 2.7 kg is at rest on the beam at a distance $L/4$ from its left end.
Calculate the scales readings:

$$F_{\text{net},y} = F_\ell + F_r - Mg - mg = 0 \qquad \text{(eq. 1)}$$

We choose to calculate the torque with respect to an axis through the left end of the beam (point O).

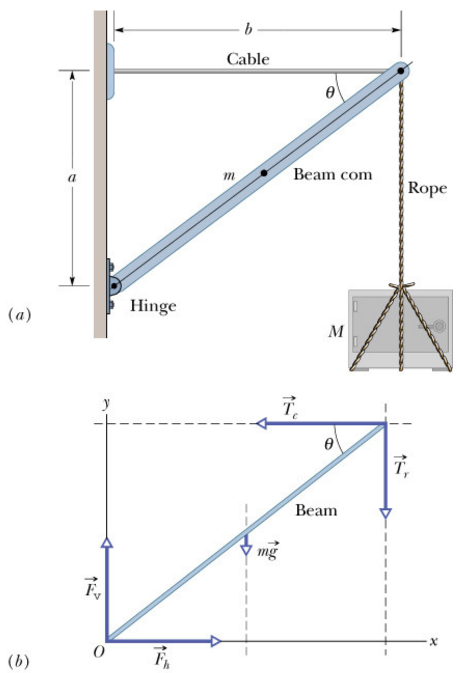
$$\tau_{\text{net},z} = -\left(\frac{L}{4}\right)(mg) - \left(\frac{L}{2}\right)(Mg) + (L)(F_r) = 0 \qquad \text{(eq. 2)}$$

From equation 2 we get: $F_r = \dfrac{Mg}{4} + \dfrac{mg}{2} = \dfrac{2.7\times 9.8}{4} + \dfrac{1.8\times 9.8}{2} = 15.44 \approx 15\text{ N.}$

We solve equation 1 for $F_\ell \rightarrow F_\ell = Mg + mg - F_r = (2.7+1.8)\times 9.8 - 15.44 = 28.66\text{ N:}$

$F_\ell \approx 29\text{ N.}$

Example – Suspended Weight



Sample Problem 12-3. A safe of mass $M = 430$ kg hangs by a rope from a boom with dimensions $a = 1.9$ m and $b = 2.5$ m. The beam of the boom has mass $m = 85$ kg. Find the tension T_c in the cable and the magnitude of the net force F exerted on the beam by the hinge.

We calculate the net torque about an axis normal to the page that passes through point O.

$$\tau_{\text{net},z} = (a)(T_c) - (b)(T_r) - \left(\frac{b}{2}\right)(mg) = 0 \rightarrow$$

$$T_c = \frac{gb\left(M + \frac{m}{2}\right)}{a} = \frac{9.8 \times 2.5(430 + 85/2)}{1.9} \approx 6100 \text{ N}$$

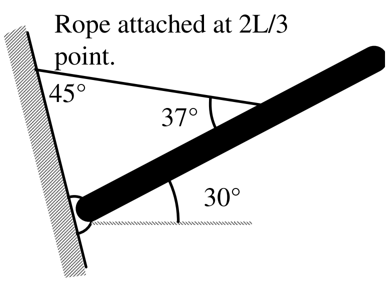
$$F_{\text{net},x} = F_h - T_c = 0 \rightarrow F_h = T_c = 6093 \text{ N}$$

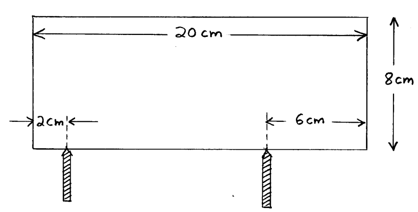
$$F_{\text{net},y} = F_v - mg - T_r = 0 \rightarrow F_v = mg + T_r = g(m + M) = 9.8 \times (85 + 430) = 5047 \text{ N}$$

$$F = \sqrt{F_h^2 + F_v^2} = \sqrt{(6093)^2 + (5047)^2} \approx 7900 \text{ N}$$

Activity – Forces on Extended Bodies

- **Concepts**
  - What is equilibrium?
  - Drawing FBDs
- **Applications**
  - Engineering problems



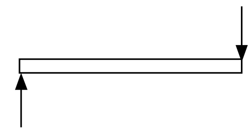


Your Name (**Print**): ______________________ Date: ________
Group Members: ______________________ Group: ________

Forces on Extended Bodies (Statics)

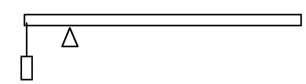
In University Physics I we concentrated on forces that can be considered to act at the center of mass of an object, and we basically treated the object as a point. Now we consider that forces on an extended object (not just an idealized point) might also rotate the object.

1. ***Equilibrium?*** A stick is shown that has only two forces acting on it, of equal magnitude and in the directions shown. (We have temporarily turned gravity off for simplicity.) Is the stick in equilibrium or not? Explain your answer.



2. ***Free Body diagram for an extended object.***

Gravity is back on again. Take a meter stick and place or attach a mass on the left end. Balance it either with your finger or on a ruler edge.



Describe in words what you would look for to know that the stick is in static equilibrium.

Draw a free body diagram for the meter stick, indicating **all** forces that act on the stick. Use the value of the suspended mass and its position, along with the balance position to calculate the mass of the meter stick. Assume the meter sick has a uniform distribution of mass. Be sure to write all relevant data below.