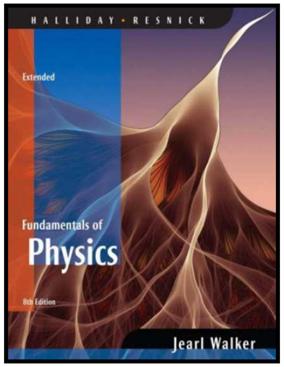
Workshop Physics 1017 - 312

University Physics II

Week 4 : Day 3

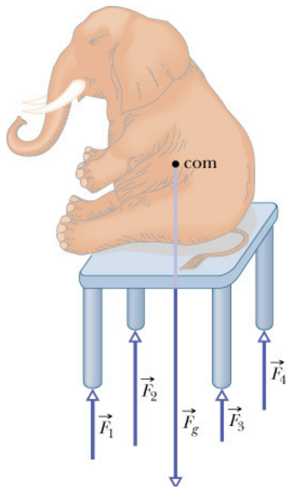


RIT

Outline

- **Indeterminate Structures**
  - Deformation of a "rigid" body
  - Defining elasticity
- **Stress and Strain in General**
  - Young's modulus – Tension and compression
  - Shear modulus – A shearing force
  - Bulk modulus – Hydraulic or compressive stress
- **A Closer Look at Tensile Stress**
  - Instron Tests
- **Activity - Linear Elasticity of Materials**
  - Examining Tensile Stress in an Elastic Cord

Indeterminate Structures



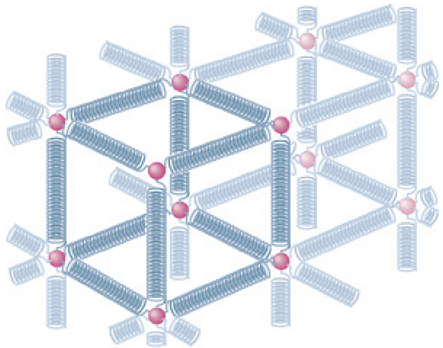
For the problems in this chapter we have the following three equations at our disposal:

$$F_{\text{net},x} = 0 \qquad F_{\text{net},y} = 0 \qquad \tau_{\text{net},z} = 0$$

If the problem has more than three unknowns we cannot solve it. We can solve a statics problem for a table with three legs but not for one with four legs. Problems like these are called indeterminate.

An example is given in the figure. A big elephant sits on a wobbly table. If the table does not collapse it will deform so that all four legs touch the floor. The upward forces exerted on the legs by the floor assume definite and different values. How can we calculate the values of these forces? To solve such an indeterminate equilibrium problem we must supplement the three equilibrium equations with some knowledge of elasticity, the branch of physics and engineering that describes how real bodies deform when forces are applied to them.

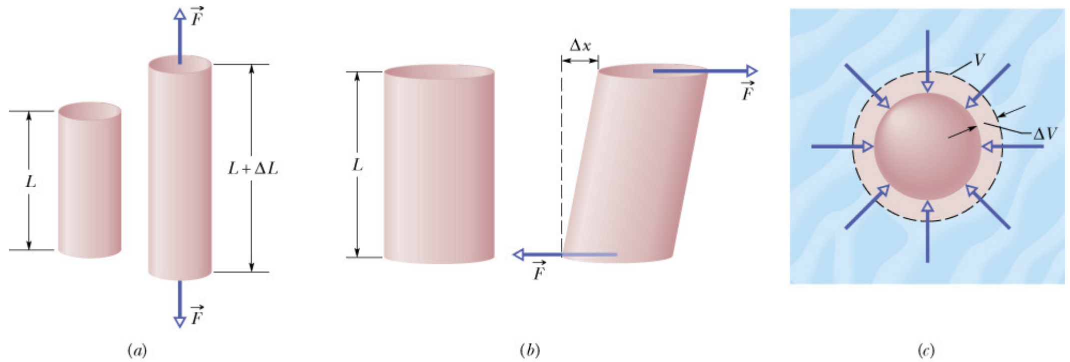
Elasticity



Metallic solids consist of a large number of atoms positioned on a regular three-dimensional lattice as shown in the figure. The lattice is repetition of a pattern (in the figure this pattern is a cube).

Each atom of the solid is a well-defined equilibrium distance from its nearest neighbors. The atoms are held together by interatomic forces that can be modeled as tiny springs. If we try to change the interatomic distance the resulting force is proportional to the atom displacement from the equilibrium position. The spring constants are large and thus the lattice is remarkably rigid. Nevertheless all "rigid" bodies are to some extent elastic, which means that we can change their dimensions slightly by pulling, pushing, twisting, or compressing them.

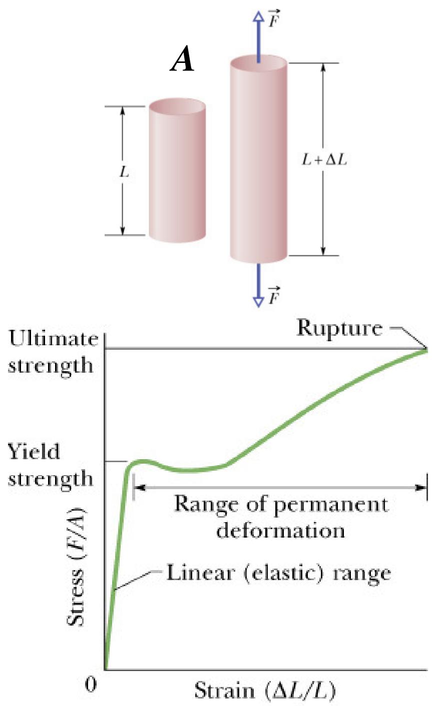
Stress and Strain



In the three figures above we show the three ways in which a solid might change its dimensions under the action of external deforming forces. In fig. *a* the cylinder is stretched by forces acting along the cylinder axis. In fig. *b* the cylinder is deformed by forces perpendicular to its axis. In fig. *c* a solid placed in a fluid under high pressure is compressed uniformly on all sides. All three deformation types have stress in common (defined as deforming force per unit area). These stresses are known as tensile/compressive for fig. *a*, shearing for fig. *b*, and hydraulic for fig. *c*. The application of stress on a solid results in strain, which takes different form for the three types of strain. Strain is related to strain

$$\text{stress} = \text{modulus} \times \text{strain}$$

Tensile Stress



Tensile stress is defined as the ratio $\frac{F}{A}$ where A is the solid area.

Strain (symbol S) is defined as the ratio $\frac{\Delta L}{L}$ where ΔL is the change in the length L of the cylindrical solid.

Stress is plotted versus strain in the upper figure.

For a wide range of applied stresses the stress-strain relation is linear and the solid returns to its original length when the stress is removed. This is known as the elastic range. If the stress is increased beyond a maximum value known as the yield strength S_y the cylinder becomes permanently deformed. If the stress continues to increase, the cylinder breaks at a stress value known as ultimate strength S_u.

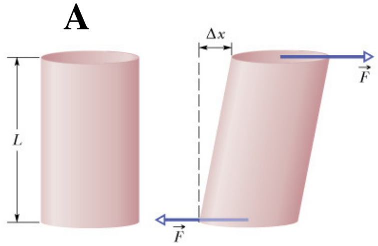
For stresses below S_y (elastic range) stress and strain are connected via the equation

$\frac{F}{A} = E\frac{\Delta L}{L}$. The constant E (modulus) is known as Young's modulus.

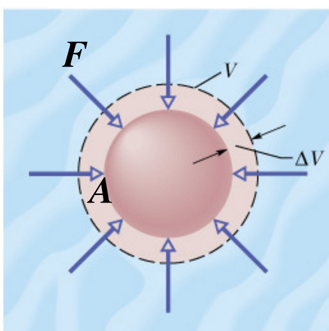
Note: Young's modulus is almost the same for tension and compression.

The ultimate strength S_u may be different.

Shear and Bulk Modulus



$$\frac{F}{A} = G\frac{\Delta x}{L}$$



$$p = B\frac{\Delta V}{V}$$

Shearing. In the case of shearing deformation, strain is defined as the dimensionless ratio $\frac{\Delta x}{L}$. The stress/strain equation has the form:

$$\frac{F}{A} = G\frac{\Delta x}{L}$$

The constant G is known as the shear modulus.

Hydraulic Stress. The stress is this case is the pressure $p = \frac{F}{A}$ that the surrounding fluid exerts on the immersed object. Here A is the area of the object. In this case strain is defined as the dimensionless ratio $\frac{\Delta V}{V}$ where V is the volume of the object and ΔV the change in the volume due to the fluid pressure. The stress/strain equation has the form: $p = B\frac{\Delta V}{V}$. The constant B is known as the bulk modulus of the material.

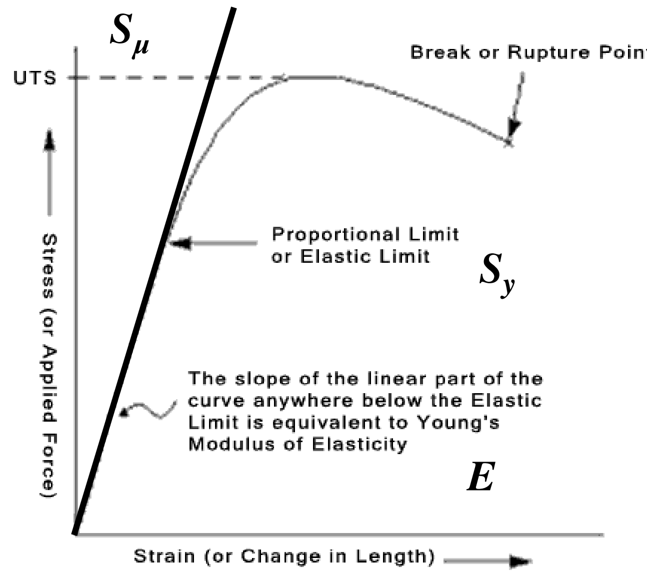
A Closer Look at Tensile Stress

- **Tensile testing of materials***
  - During the initial portion of the test, the relationship between the applied force, or load, and the elongation the specimen exhibits is linear.
  - In this linear region, the line obeys the relationship defined as "Hooke's Law" where the ratio of stress to strain is a constant:

$$\underbrace{\frac{F}{A}}_{STRESS} = E\,\underbrace{\frac{\Delta L}{L}}_{STRAIN} \Rightarrow \frac{F/A}{\Delta L/L} = E$$

  - At the point that the curve is no longer linear, Hooke's Law no longer applies and some permanent deformation occurs in the specimen.
  - A value called "yield strength" of a material is defined as the stress applied to the material at which plastic deformation starts to occur while the material is loaded.
  - One of the properties you can determine about a material is its ultimate tensile strength (UTS).
    - *Maximum load the specimen sustains during the test.*
    - *The UTS may or may not equal the strength at break.*

Tensile Stress Testing



True Strain: $\varepsilon \equiv \ln\left(\frac{L_i}{L_0}\right)$

Material adapted from Instron site: http://www.instron.us/wa/applications/test_types/tension/default.aspx

Elastic Properties of Materials

- See table 12-1

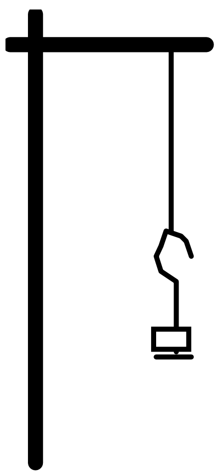
View ***INSTRON*** *
test on *YouTube*

Material	Bulk Density ρ (kg/m³)	Young's Modulus E (10^9 N/m²)	Ultimate Strength S_μ (10^6 N/m²)	Yield Strength S_y (10^6 N/m²)
Steel	7860	200	400	250
Aluminum	2710	70	110	95
Glass	2190	65	50	—
Concrete	2320	30	40	—
Wood	525	13	50	—
Bone	1900	9	170	—
Polystyrene	1050	3	48	—

*If above link does not work copy and paste the following URL in your browser: http://www.youtube.com/watch?v=CkSsRLUeRQI

Activity – Linear Elasticity of Materials

- **Examining Tensile Stress in an Elastic Cord**
  - Apply various weights
  - Determine the force:
    - *Use mg*
  - Determine elongation
    - *Use Δx*
- **Hooke's Law and Tensile Stress**
  - Plot data and discover elasticity for the cord.
  - When is Hooke's Law not appropriate?
    - *What is yield strength of cord?*



$$F = -k\Delta x$$

Your Name (**Print**): ____________ Date: ________
Group Members: ____________ Group: ________

Linear Elasticity of Materials

You are already familiar with Hooke's Law for a spring, $F = -k\,\Delta x$, where Δx measures the stretch of the spring from its relaxed state. Let's first quickly review this experimentally, as we'll be using springs in the upcoming labs on simple harmonic motion, and it's important to understand the distinction between the position x and the extension Δx.

Hang the *small end* (can you think of why the small end?) of the spring from the support and measure the Δx extension of the spring in some reasonable manner. Consider the relaxed state to be when the mass holder alone is on the cord, and use this as the zero of your force. The mass holder may stretch the spring a little bit, but this can still be taken as $\Delta x = 0$ providing the plot in a linear relationship and we consider the force to be the weight added to the holder.

Data Table 1: Now add the following masses and measure the length of the spring, including uncertainties.

Mass added to holder (grams)	Length of spring (cm)	Elongation Δx (cm)
0		
5		
10		
15		
30		
60		
80		
100		
150		
200		
250		
300 (Don't exceed this for the spring!)		

(Substitute other masses here if you can't arrange the ones shown, just don't exceed 300 grams).