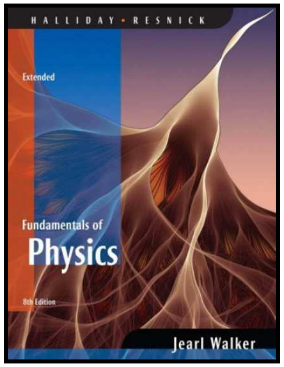
Workshop Physics 1017 - 312

University Physics II

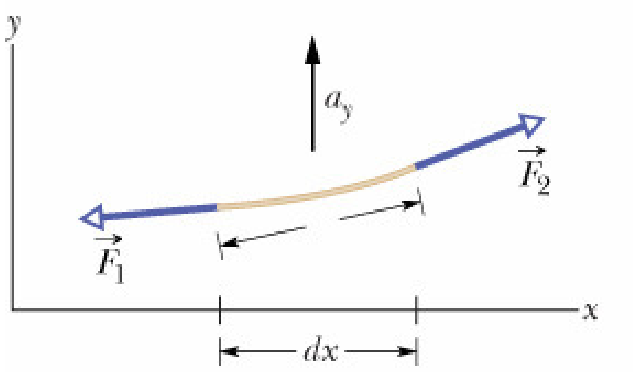


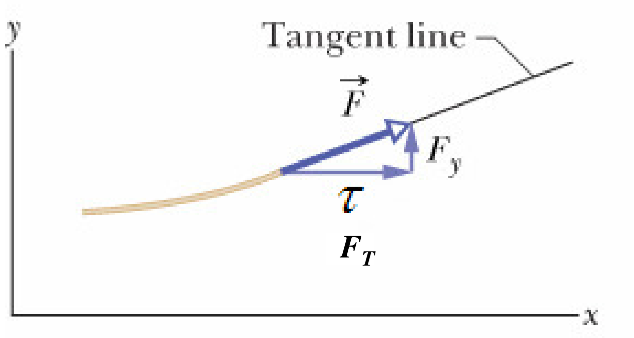
Week 7 : Day 1

Outline

- **Standing waves**
  - The wave equation
  - Speed of propagation
  - Resonance conditions
  - Frequency harmonics
- **Activity**
  - Standing waves on a string

The Wave Equation





$$\frac{F_y}{F_T} = \frac{dy}{dx}$$

Consider a string of mass density μ and tension τ.

A transverse wave propagates along the string.

The transverse motion is described by $y(x,t)$.

Consider an element of length dx and mass $dm = \mu dx$.

From Newton's second law we have:

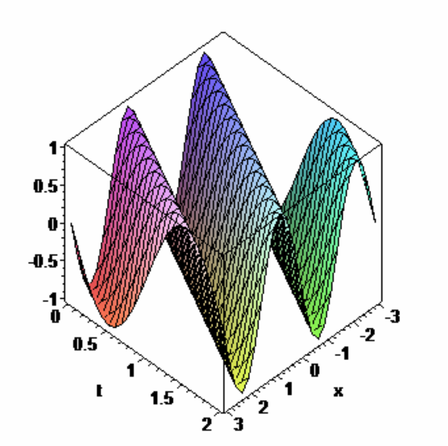
$$dF_y = dma_y \Rightarrow \frac{d}{dx}F_y = \frac{dm}{dx}\left(\frac{d^2y}{dt^2}\right)$$

$$\Rightarrow \frac{d}{dx}\left(F_T\frac{dy}{dx}\right) = \mu\frac{d^2y}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{\mu}{F_T}\frac{d^2y}{dt^2}}$$

Adapted from Halliday, Resnick and Walker; © Michael Jay Schillaci

Wave Speed on a Stretched String



From the wave equation we can find the velocity of the wave directly as follows:

$$\frac{d^2y}{dx^2} = \frac{\mu}{F_T}\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\frac{dy}{dt}\frac{dt}{dx} = \frac{1}{v}\frac{d}{dx}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{1}{v}\frac{dt}{dx}\frac{d}{dt}\frac{dy}{dt} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} = C\frac{d^2y}{dt^2}$$

$$\Rightarrow \frac{1}{v^2} = \frac{\mu}{F_T} \Rightarrow v = \sqrt{\frac{F_T}{\mu}}$$

One solution to the above differential equation is given by a simple sin function in the following form :

$$\Rightarrow y(x,t) = y_m \sin(kx - \omega t + \phi)$$

Standing Wave Model

- ☐ **Consider two waves traveling in opposite directions:**

$$y'(x,t) = \underbrace{y_m \sin(kx - \omega t)}_{traveling \quad right} + \underbrace{y_m \sin(kx + \omega t)}_{traveling \quad left}$$

- ☐ **Use double angle formulae:**

$$\sin(kx - \omega t) = \sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)$$

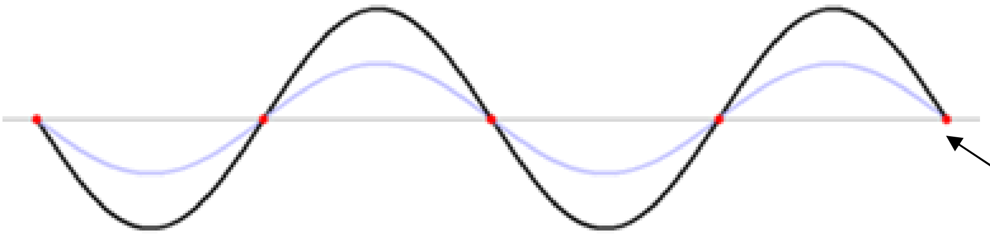
$$\sin(kx + \omega t) = \sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t)$$

- ☐ **Add the resulting waves:**

$$y'(x,t) = 2y_m \sin(kx)\cos(\omega t)$$

Note: This is not a harmonic wave!

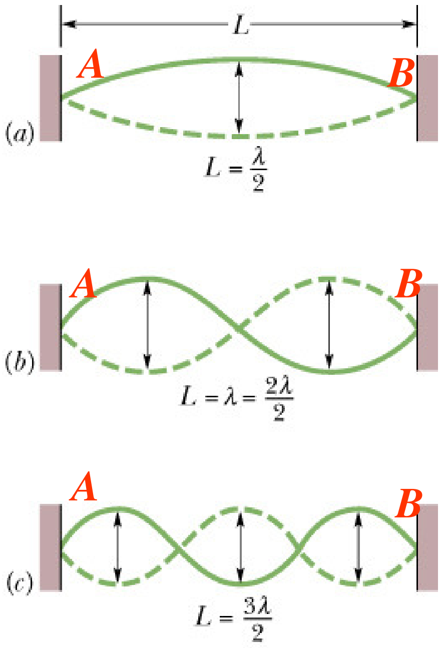
A Standing Wave is a sum of two harmonic waves traveling in opposite directions…



The stationary points or "nodes" of a Standing wave do not move

Picture credit: http://en.wikipedia.org/wiki/Standing_wave

Resonance conditions



Resonances occur when the resulting standing wave satisfies the boundary condition of the problem. These are that the amplitude must be zero at point A and point B and arise from the fact that the string is clamped at both points and therefore cannot move. The first resonance is shown in fig. a. The standing wave has two nodes at points A and B. Thus $L = \frac{\lambda_1}{2} \rightarrow \lambda_1 = 2L$. The second standing wave is shown in fig. b. It has three nodes (two of them at A and B). In this case $L = 2\left(\frac{\lambda}{2}\right) = \lambda \rightarrow \lambda_2 = L$.

The third standing wave is shown in fig. c. It has four nodes (two of them at A and B). In this case $L = 3\left(\frac{\lambda}{2}\right) = \lambda \rightarrow \lambda_3 = \frac{2}{3}L$. The general expression for the resonant wavelengths is $\lambda_n = \frac{2L}{n}$ for $n = 1, 2, 3, \ldots$.

$$\boxed{\text{The resonant frequencies } f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}.}$$

Nodes and Antinodes

- The superposition of the two waves
  - Is not a harmonic wave
  - Forms a "standing wave"

$$y'(x,t) = \underbrace{2y_m \sin(kx)}_{Amplitude} \cos(\omega t)$$

- Locating Nodes and Antinodes

Nodes:

$$kx_n = \frac{2\pi}{\lambda_n} x_n = n\pi$$

$$\Rightarrow x_n = n\frac{\lambda_n}{2}$$

$$\Rightarrow = (2n)\frac{\lambda_n}{4}, \qquad n = 0,1,2,3\ldots$$

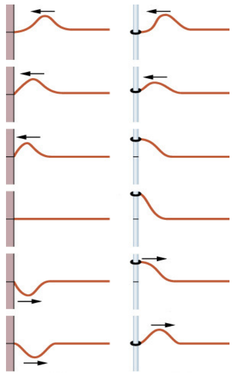
Antinodes:

$$kx_n = \frac{2\pi}{\lambda_n} x_n = \left(n + \frac{1}{2}\right)\pi$$

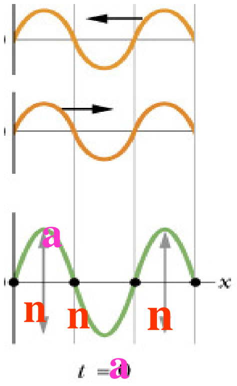
$$\Rightarrow x_n = \left(n + \frac{1}{2}\right)\frac{\lambda_n}{2}$$

$$\Rightarrow x_n = (2n+1)\frac{\lambda_n}{4}, \qquad n = 0,1,2,3\ldots$$

Reflection at a Boundary



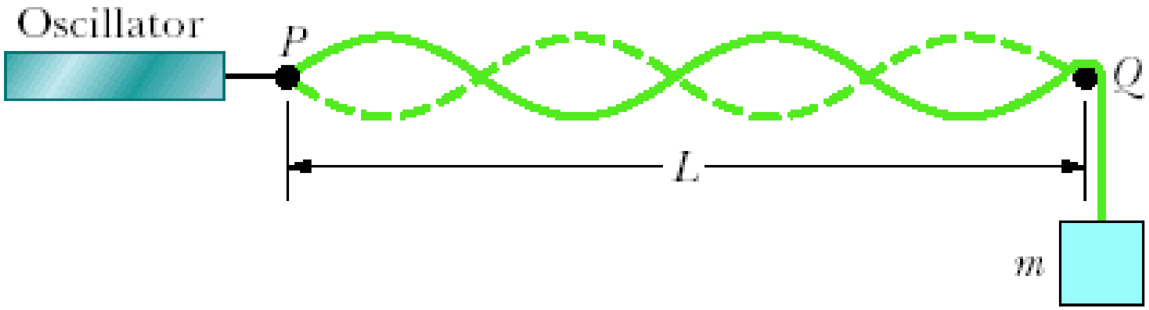
Standing Wave Formation



Standing Waves on a String

- ☐ **Consider an oscillator system**
  - Variable frequency generator
  - Extensible string, mass and hanger
  - Various clamps and pulleys

$$v = \sqrt{\frac{F_T}{\mu}}$$



Generator Frequency

$$v = f\lambda$$

$$\Rightarrow f_n = \frac{v}{\lambda_n}$$

Extensible String

$$\mu_0 \equiv \frac{m}{L_0} \Rightarrow \mu_{stretched} \equiv \frac{L_{unstretched}}{L_{stretched}} \mu_{unstretched}$$

$$\Rightarrow \mu_s = \frac{L_u}{L_s} \mu_0$$

Applied Force

$$F_T = m_{applied}\, g$$

$$\Rightarrow F_T = \left(m_{added} + m_{hanger}\right) g$$

Adapted from Halliday, Resnick and Walker; © Michael Jay Schillaci

Frequency Harmonics

- ☐ **Use velocity to predict the frequency harmonics**

$$v = \sqrt{\frac{F_T}{\mu}} = f\lambda$$

Applied Tension

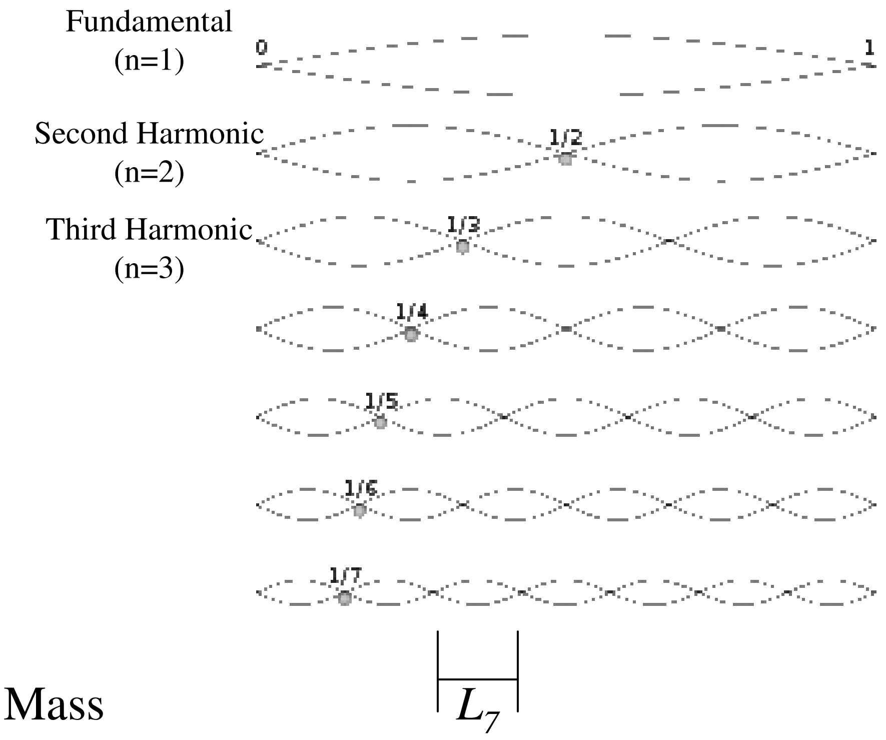
$$\Rightarrow f_n = \frac{v}{\lambda_n} = \frac{\sqrt{\frac{F_T}{\mu}}}{\frac{2L}{n}} = \frac{1}{2L_n}\sqrt{\frac{mg}{\mu_s}}$$

Resonance Condition

Node Length

Stretched Mass Density

Standing Wave Patterns



Length Between Nodes → L_n

Picture credits: http://en.wikipedia.org/wiki/Standing_wave

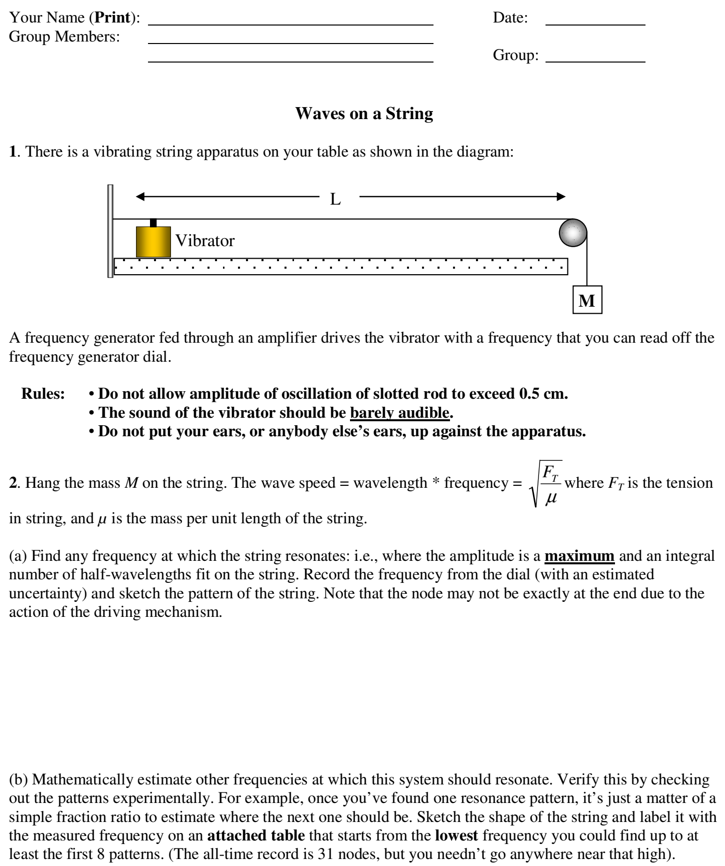
Activity – Waves on a String

- ☐ **Find the frequency at which the string resonates at its' fundamental for the applied tension value**
- ☐ **Calculate the theoretical resonant frequencies for the patterns**

$$f_n = \frac{v}{\lambda_n}$$

- ☐ **Sketch the shape of the string and label it with the measured frequency**
  - ➢ Make an **table** that starts from the **lowest** frequency you could find up to <u>at least</u> the first 8 patterns
- ☐ **Calculate the uncertainties in your theoretical frequency values**
  - ➢ Determine whether he theoretical and experimental frequencies agree within uncertainty
  - ➢ Calculate a % error for each predicted frequency

$$\%Error = \frac{\left| f_{n_{Experimental}} - f_{n_{Theoretical}} \right|}{\frac{\left(f_{n_{Experimental}} - f_{n_{Theoretical}} \right)}{2}} \times 100$$



Your Name (**Print**): ______________ Date: ______
Group Members: ______________
______________ Group: ______

Waves on a String

1. There is a vibrating string apparatus on your table as shown in the diagram:

A frequency generator fed through an amplifier drives the vibrator with a frequency that you can read off the frequency generator dial.

Rules:
- **Do not allow amplitude of oscillation of slotted rod to exceed 0.5 cm.**
- **The sound of the vibrator should be <u>barely audible</u>.**
- **Do not put your ears, or anybody else's ears, up against the apparatus.**

2. Hang the mass *M* on the string. The wave speed = wavelength * frequency = $\sqrt{\frac{F_T}{\mu}}$ where F_T is the tension in string, and μ is the mass per unit length of the string.

(a) Find any frequency at which the string resonates: i.e., where the amplitude is a **<u>maximum</u>** and an integral number of half-wavelengths fit on the string. Record the frequency from the dial (with an estimated uncertainty) and sketch the pattern of the string. Note that the node may not be exactly at the end due to the action of the driving mechanism.

(b) Mathematically estimate other frequencies at which this system should resonate. Verify this by checking out the patterns experimentally. For example, once you've found one resonance pattern, it's just a matter of a simple fraction ratio to estimate where the next one should be. Sketch the shape of the string and label it with the measured frequency on an **attached table** that starts from the **lowest** frequency you could find up to at least the first 8 patterns. (The all-time record is 31 nodes, but you needn't go anywhere near that high).