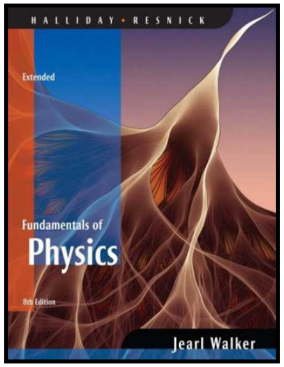
Workshop Physics 1017 - 312

University Physics II

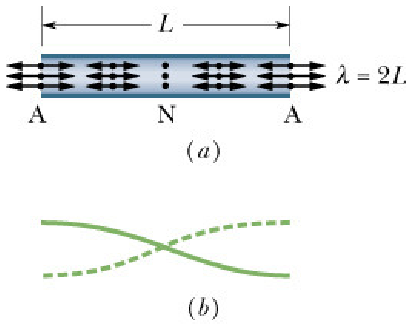


Week 7 : Day 3

Outline

- **Standing waves in a pipe**
  - Open at both ends
  - Closed at one end and open at other end
- **Interference of sound waves**
  - Path difference and phase difference
  - Constructive and deconstructive interference
- **Adding sound waves**
  - The Beat Frequency
    - *Frequency range, amplitude dependence*
- **Activities**
  - Activity - Beat Frequency
  - Activity - The Fourier Transform

Standing Waves in a Pipe



Consider a pipe filled with air that is open at both ends. Sound waves that have wavelengths that satisfy a particular relation with the length L of the pipe set up standing waves that have sustained amplitudes.

The simplest pattern can be set up in a pipe that is open at both ends as shown in fig.a. In such a pipe, standing waves have an antinode (maximum) in the dispacement amplitude. The amplitude of the standing wave is plotted as a function of distance in fig. b. The pattern has a node at the pipe center since two adjacent antinodes are separated by a node (minimum). The distance between two adjacent antinodes is $\lambda/2$.

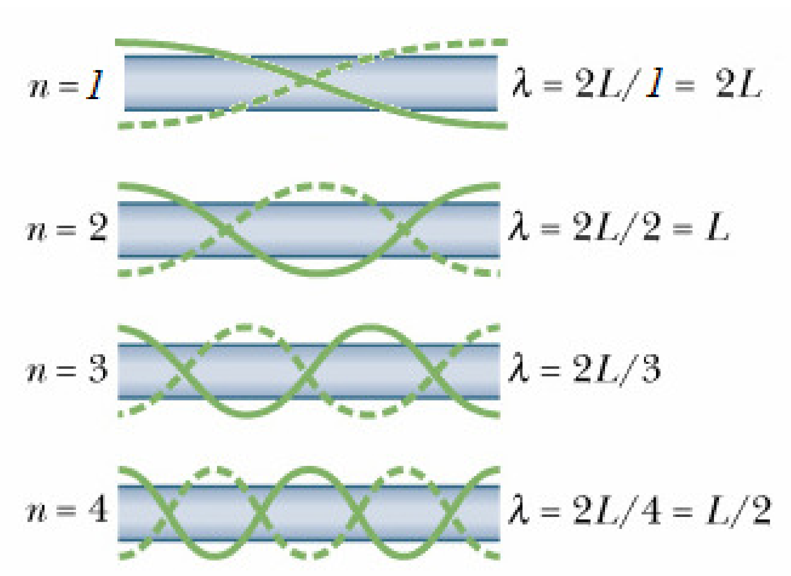
Thus $L = \lambda / 2 \rightarrow \lambda = 2L$. Its frequency $f = \dfrac{v}{\lambda} = \dfrac{v}{2L}$.

The standing wave of fig. b is known as the "***fundamental mode***" or "***first harmonic***" of the tube.

Note : Antinodes in the displacement amplitude correspond to nodes in the pressure amplitude. This is because s_m and Δp_m are 90° out of phase.

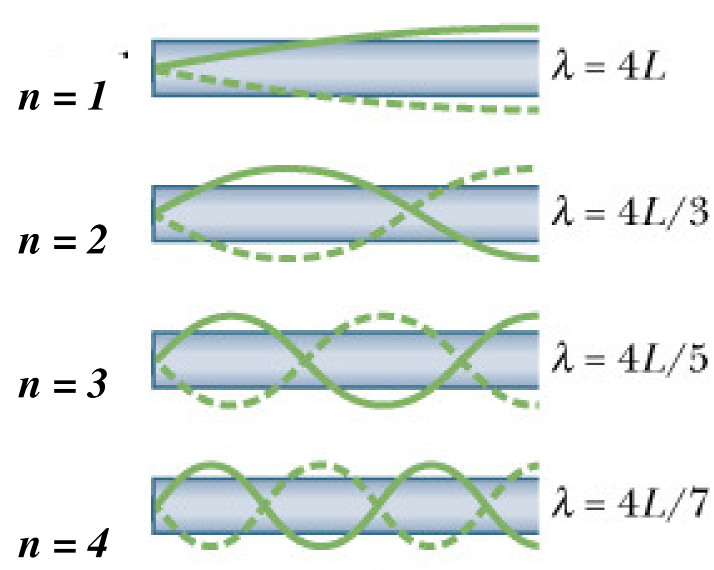
Open and Closed Pipes

Standing Waves in Pipes Open at Both Ends



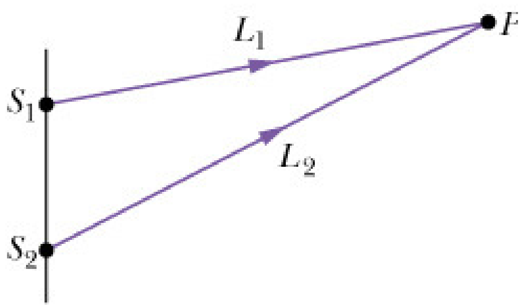
$$\lambda_n = \frac{2L}{n}$$

Standing Waves in Pipes Open at One End and Closed at the Other



$$\lambda_n = \frac{2L}{n - \frac{1}{2}}$$

Wave Interference



Consider two point sources of sound waves S_1 and S_2 shown in the figure. The two sources are in phase and emit sound waves of the same frequency.

Waves from both sources arrive at point P whose distance from S_1 and S_2 is L_1 and L_2, respectively.

The two waves interfere at point P.

At time t the phase of sound wave 1 arriving from S_1 at point P is $\phi_1 = kL_1 - \omega t$.

At time t the phase of sound wave 2 arriving from S_2 at point P is $\phi_2 = kL_2 - \omega t$.

In general, the two waves at P have a phase difference

$$\phi = |\phi_2 - \phi_1| = |kL_2 - \omega t - (kL_1 - \omega t)| = k|L_2 - L_1| = \frac{2\pi}{\lambda}|L_2 - L_1|.$$

$$\Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\Delta L \qquad \Rightarrow \qquad \frac{\Delta\phi}{\Delta L} = \frac{2\pi}{\lambda}$$

Phase Difference ($\Delta\phi$)

Path Difference (ΔL)

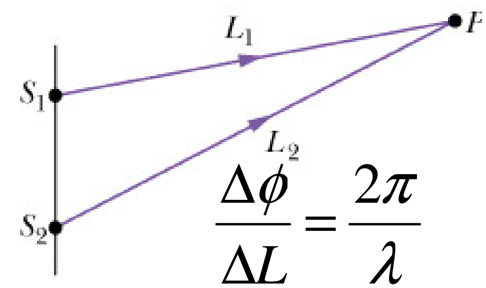
Interference Conditions

Constructive Interference

The wave at P resulting from the interference of the two waves that arrive from S_1 and S_2 has a maximum amplitude when the phase difference $\phi = 2\pi m$ for

$m = 0, 1, 2, \ldots \; . \;\; \rightarrow \dfrac{2\pi}{\lambda}\Delta L = 2\pi m \;\; \rightarrow \Delta L = m\lambda$ where

$\Delta L = 0,\ \lambda,\ 2\lambda, \ldots$



$$\frac{\Delta\phi}{\Delta L} = \frac{2\pi}{\lambda}$$

ΔL equal to an integral multiple of λ → **constructive interference**

Destructive Interference

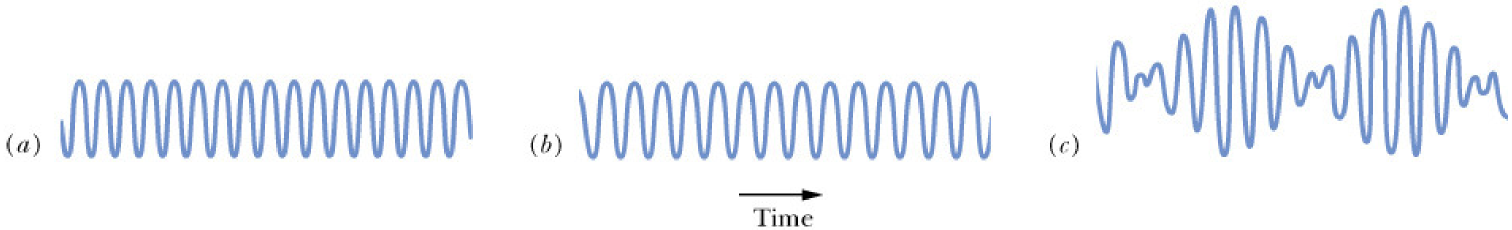
The wave at P resulting from the interference of the two waves that arrive from S_1 and S_2 has a minimum amplitude when the phase difference

$\phi = \pi(2m+1)$ for $m = 0, 1, 2, \ldots \; . \;\; \rightarrow \dfrac{2\pi}{\lambda}\Delta L = \pi(2m+1) \;\; \rightarrow$

$\Delta L = \left(m + \dfrac{1}{2}\right)\lambda$ where $\Delta L = \lambda/2,\ 3\lambda/2,\ 5\lambda/2, \ldots$

ΔL equal to a half-integral multiple of λ → **destructive interference**

Adding Sound Waves

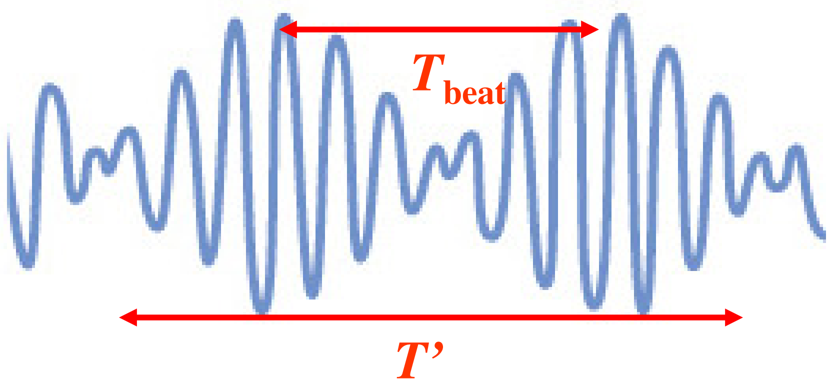


If we listen to two sound waves of equal amplitude and frequencies f_1 and f_2 $\left(f_1 > f_2 \text{ and } f_1 \approx f_2\right)$ we perceive them as a sound of frequency $f_{av} = \dfrac{f_1 + f_2}{2}$. In addition we also perceive "beats," which are variations in the intensity of the sound with frequency $f_{\text{beat}} = f_1 - f_2$. The displacements of the two sound waves are given by the equations $s_1 = s_m \cos \omega_1 t$ and $s_2 = s_m \cos \omega_2 t$. These are plotted in fig. a and fig. b.

Using the principle of superposition we can determine the resultant displacement as

$$s = s_1 + s_2 = s_m \left(\cos \omega_1 t + \cos \omega_2 t\right) = 2 s_m \cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right) t\right] \cos\left[\left(\frac{\omega_1 + \omega_2}{2}\right) t\right]$$

The Beat Frequency



$$f_{\text{beat}} = f_1 - f_2$$

$$s = [2s_m \cos\omega' t]\cos\omega t \quad \text{where} \quad \omega' = \frac{\omega_1 - \omega_2}{2} \quad \text{and} \quad \omega = \frac{\omega_1 + \omega_2}{2}$$

The displacement s is plotted as a function of time in the figure. We can regard it as a cosine function whose amplitude is equal to $|2s_m \cos\omega' t|$.

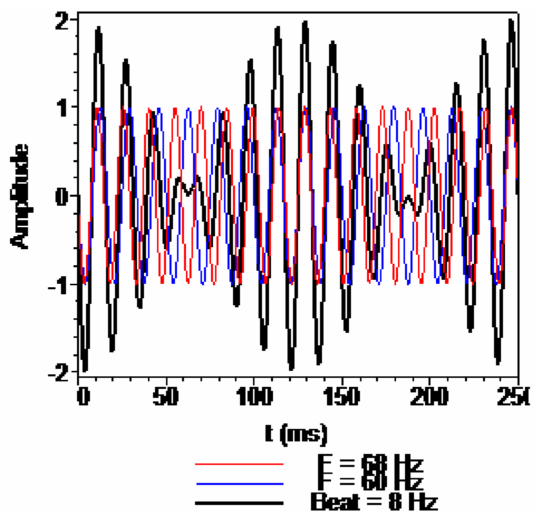
The amplitude is time dependent but varies slowly with time. The amplitude exhibits a maximum whenever $\cos\omega' t$ is equal to either +1 or -1, which happens twice within one period of the $\cos\omega' t$ function.

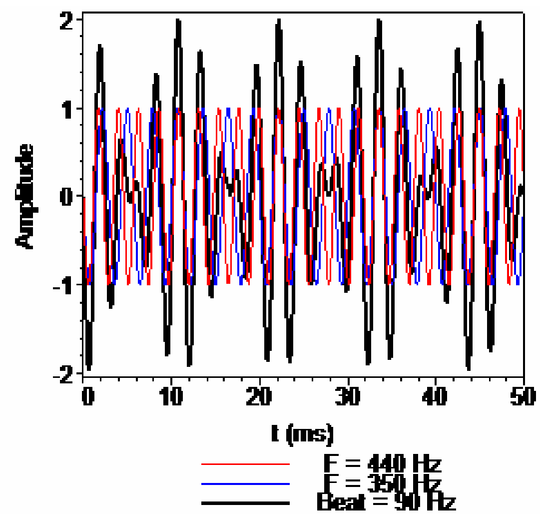
Thus the angular frequency of the beats $\omega_{\text{beat}} = 2\omega' = 2\left(\frac{\omega_1 - \omega_2}{2}\right) = \omega_1 - \omega_2$.

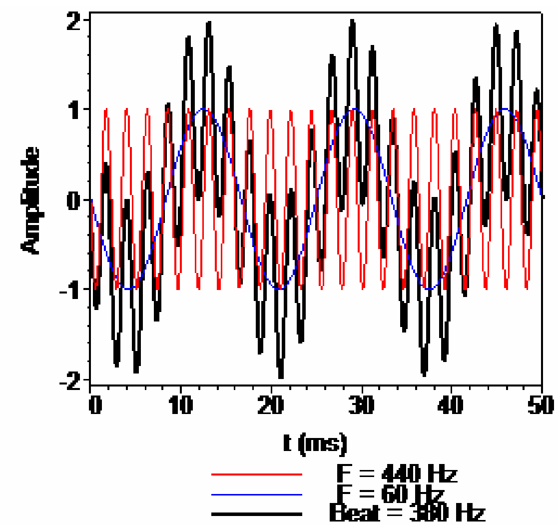
The frequency of the beats $f_{\text{beat}} = 2\pi\omega_{\text{beat}} = 2\pi\omega_1 - 2\pi\omega_2 = f_1 - f_2$.

Beat Frequency Examples

- The frequency of constituent waves matters
  - Human hearing range is 20 – 20,000 Hz



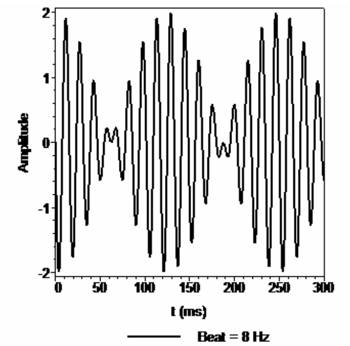




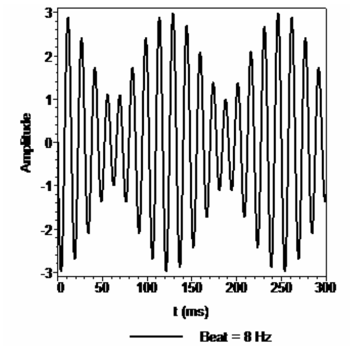
Amplitude Dependence

- **The amplitude of the constituent waves can also effect the perception of the "beat"…**
  - Consider a sound wave of the general form

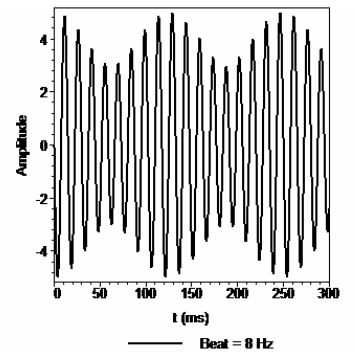
$$s_n(x,t) = A_n \sin\left[2\pi f_n\left(\frac{x}{v} - t\right) + \phi_n\right]$$



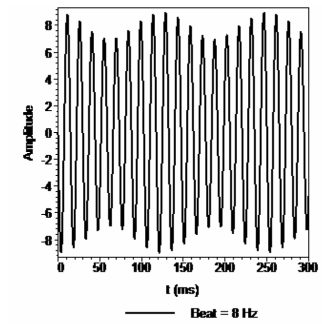
A_1=1, A_2=1



A_1=2, A_2=1



A_1=4, A_2=1

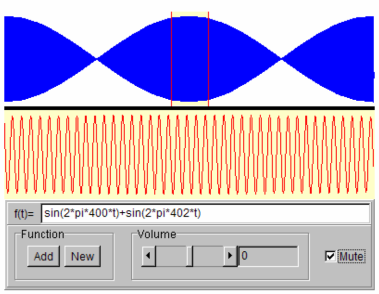


A_1=8, A_2=1

Activity – Beat Frequencies

- Combine waves using trigonometric identify to show beat frequency formula:
  - $$\sin\alpha \pm \sin\beta = 2\sin\left[\frac{\alpha\pm\beta}{2}\right]\cos\left[\frac{\alpha\mp\beta}{2}\right]$$

- Explore applets
  - Use Applet on the second page to do questions (1) – (6



Your Name (**Print**): ______________________ Date: __________

Group Members: ______________________ Group: __________

Beat Frequencies

Beats: equal amplitudes, same direction, slightly different frequencies and wavelengths

The two waves can be represented by

$$y_1 = y_m \sin(k_1 x - \omega_1 t) \quad \text{and} \quad y_2 = y_m \sin(k_2 x - \omega_2 t)$$

Since they are equal amplitude, the sum can be obtained from the trig identity:

$$\sin\alpha \pm \sin\beta = 2\sin\left[\frac{\alpha\pm\beta}{2}\right]\cos\left[\frac{\alpha\mp\beta}{2}\right]$$

Use this trig identity to combine the two waves:

Notice that the argument of the sine function contains just the average wave number and the average angular frequency, while the argument of the cosine term contains a small wave number and a small frequency (the difference between the two frequencies is small).

Link to the following website:

http://www.sciencejoywagon.com/explrsci/media/tonebeat.htm

(1) Listen to the 440 Hz and 441 Hz tones separately first. (The tone mercifully turns itself off after a second or so). Can you tell these two sounds apart? If your lab partner claims that they can hear the difference separately, make sure to test them on this by not letting them see the screen.

(2) Now listen to the two sounds together. What do you hear?

(3) Now listen to the 440 and 443 Hz sounds -- separately, together -- What do you hear?

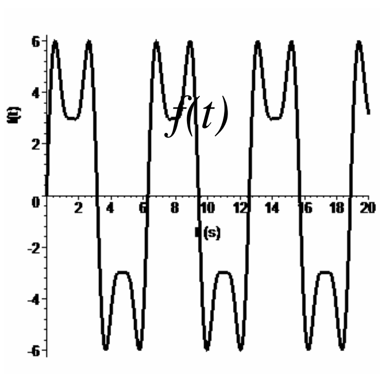
(4) How is this different from part (2)?

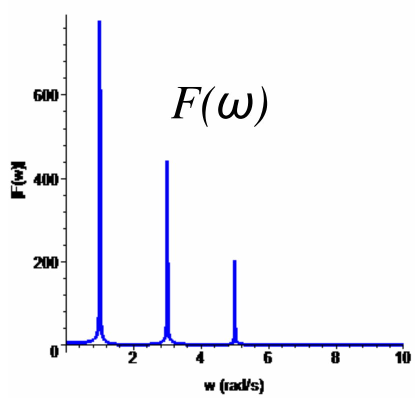
The Fourier Transform

- **Decompose a signal into its frequency spectrum**
  - Use the *Fourier Transform*:

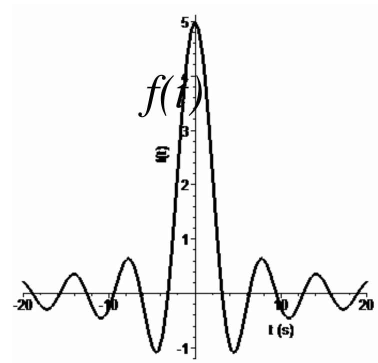
$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} \, dt$$

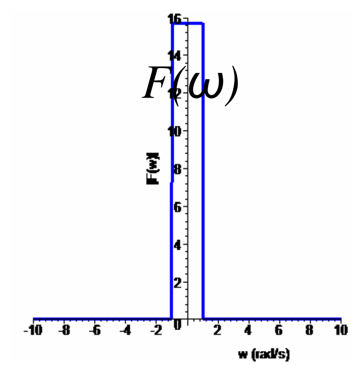
  - Examples





$$f(t) = 5\sin(t) + 3\sin(3t) + \sin(5t)$$





$$f(t) = A\frac{\sin(ct)}{ct}$$

Activity – Measuring Sound

- Recording sound waves for tuning forks
  - Single tuning fork (fit to get frequency)
  - Two tuning forks (note beat pattern)
- Take the Fourier transform of sound waves
  - Analyze to see frequency spectrum
    - *Single tuning fork*
    - *Two tuning forks*
    - *Your “singing” voice*

Your Name (**Print**): ______________________ Date: ________
Group Members: ______________________
______________________ Group: ________

Measuring Sound: Beats and Fourier Transforms

Never strike the tuning fork against a hard surface like the edge of the table, or your lab partner’s head, as this will damage the tuning fork. Instead use the striking block provided, or the heel of your hand. Do not put the tuning forks in your mouth, as the vibrations can shatter fillings.

1. Hold a tuning fork and strike it against the heel of your hand. Hold it near your ear and rotate the tuning fork. Describe what you hear, and explain why you hear what you hear.

2. Hook up the LabPro to your computer and connect the microphone to Ch 1. **Plug the power into the LoggerPro box first**. Open the Student Shares to *Team Physics 312: LoggerPro.* Copy the file *Measuring Sound minilab* to your MyDocuments folder, then drag the icon onto the LoggerPro icon to open it.

3. Check the settings on the collection (*Experiment → Data Collection*): you should be set to collect for 0.5 sec at 10000 samples per second. Zero the microphone when there is little or no sound (control zero). Test the setup by recording a nice sine wave for a single fork. (If it’s not a nice sine wave, it’s possible that particular fork was dropped or struck too hard). Check that the frequency on the computer matches the frequency labeled on the tuning fork.

4. Get two tuning forks that have different frequencies, but not too different. About 5 to 10 percent difference is good.

   Strike one of the tuning forks firmly against the heel of your hand or the striking block (not a hard surface!), hold the fork in front of the microphone and collect data for the 0.5 sec. Click and drag on the horizontal axis to scale it out.

   Make sure you can see the sinusoidal wave (if you can't, try changing the time range to 0.2 seconds). Measure the period of the sound by counting cycles on the graph and calculate the frequency from the period. Compare the frequency calculated this way to the number stamped on the tuning fork. Try to fit a sine function to the graph to check the frequency you got from the period.

5. Strike both forks and hold in front of the microphone as you collect. You will need to adjust distances, and repeat this a few times until the microphone hears both forks. What does the wave look like now? Can you tell that there are two different frequencies, and what they are?