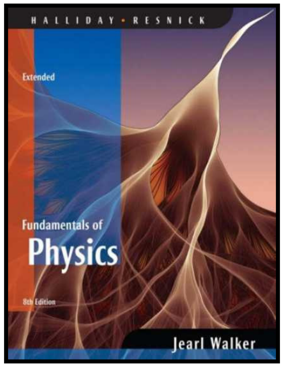
University Physics II



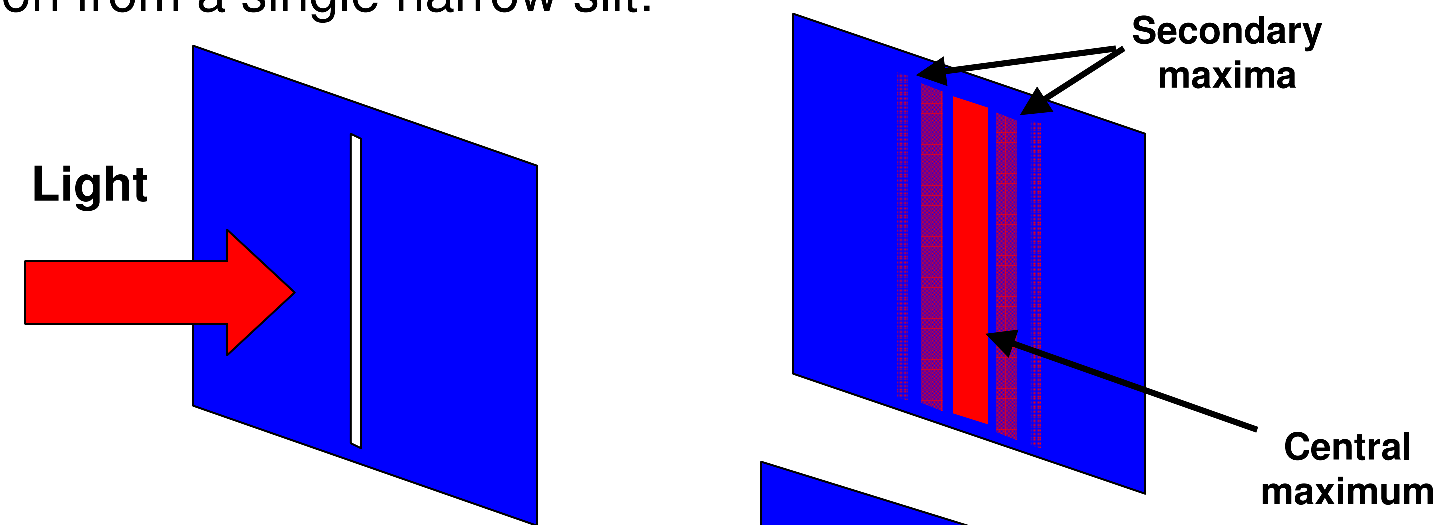
Week 9 : Day 2

Outline

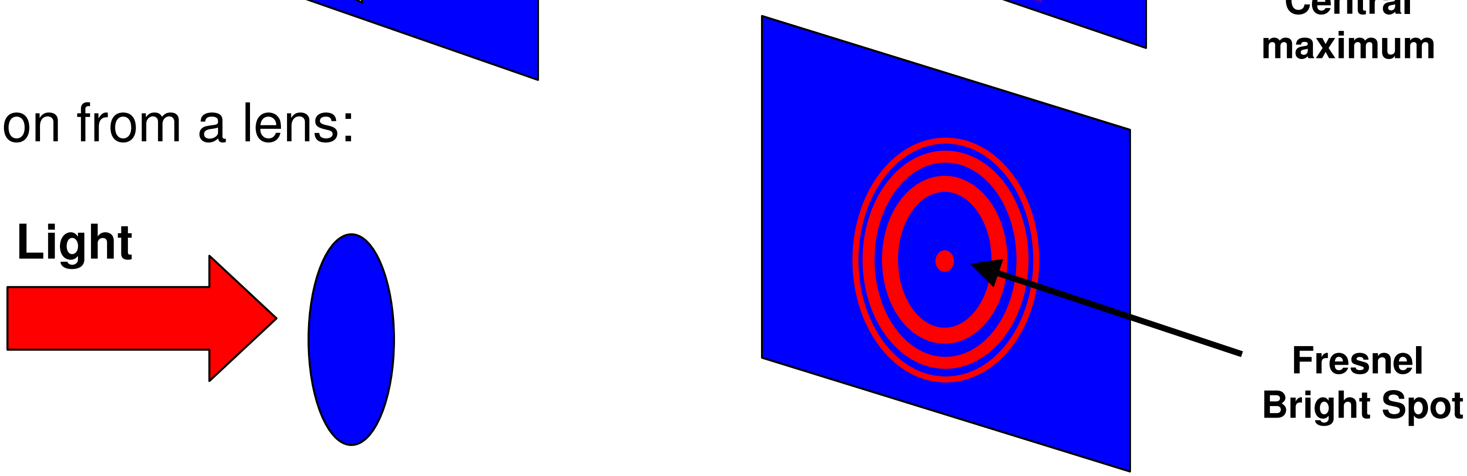
- The wave theory of light
  - Diffraction
  - Interference
- Single-slit diffraction
  - Locating diffraction minima
  - Intensity pattern
- Activity - Diffraction

Diffraction and the Wave Theory of Light

Diffraction from a single narrow slit:

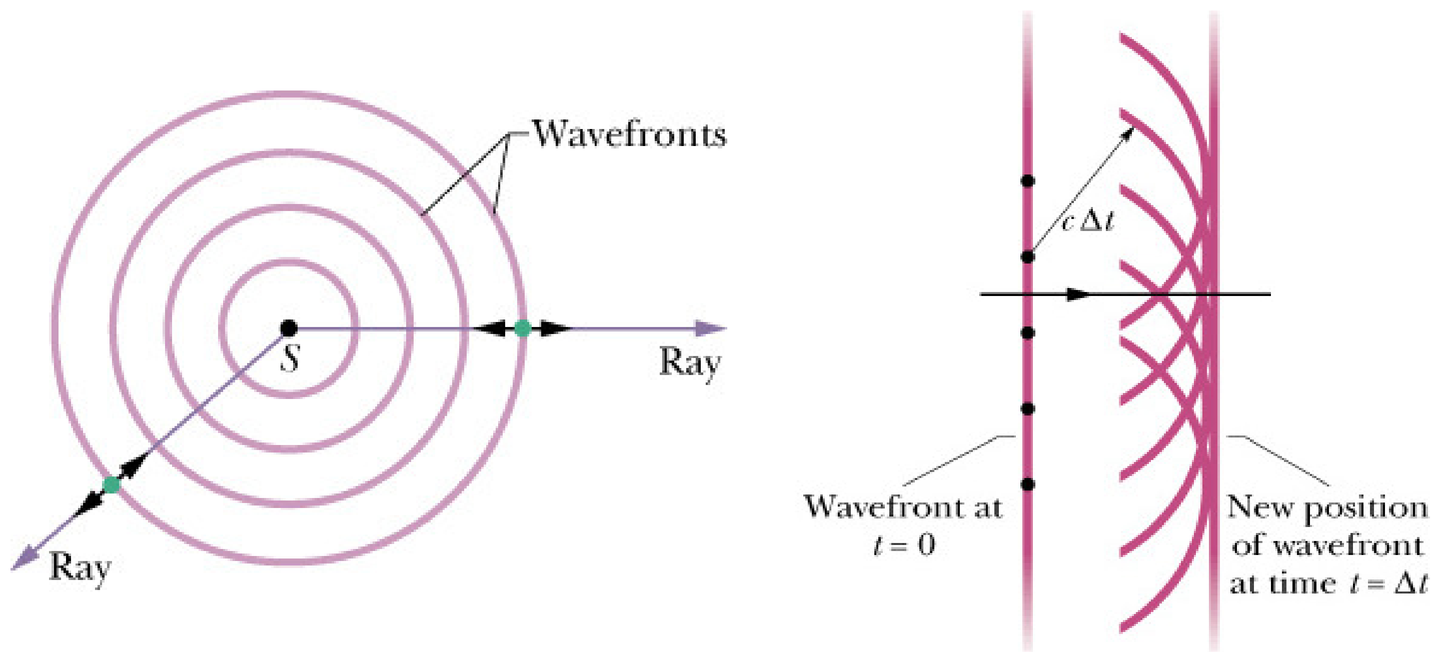


Diffraction from a lens:



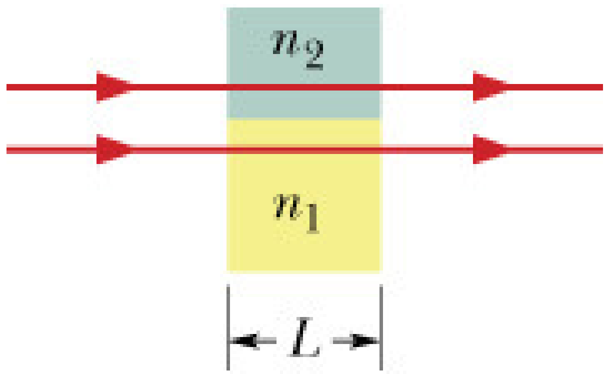
Light as a Wave

Huygen's Principle: All points on a wavefront serve as point sources of spherical secondary wavelets.



After time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

Wavelength and the Index of Refraction



The frequency of light in a medium is the same as it is in vacuum.

$$\lambda_n = \frac{\lambda}{n} \qquad f_n = \frac{v}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f$$

Since wavelengths in n_1 and n_2 are different, the two beams may no longer be in phase.

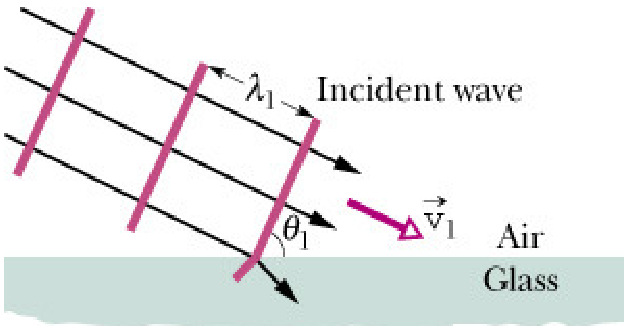
$$\text{Number of wavelengths in } n_1\text{: } N_1 = \frac{L}{\lambda_{n1}} = \frac{L}{\lambda/n_1} = \frac{Ln_1}{\lambda}$$

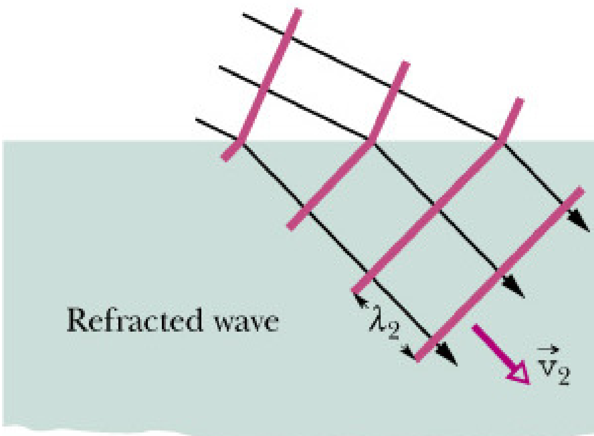
$$\text{Number of wavelengths in } n_2\text{: } N_2 = \frac{L}{\lambda_{n2}} = \frac{L}{\lambda/n_2} = \frac{Ln_2}{\lambda}$$

$$\text{Assuming } n_2 > n_1\text{: } N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_2}{\lambda} = \frac{L}{\lambda}(n_2 - n_1)$$

⇨ $N_2 - N_1 = 1/2$ wavelength $\rightarrow$ destructive interference

Wavelength and the Law of Refraction





Law of Refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Index of Refraction:

$$n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2}$$

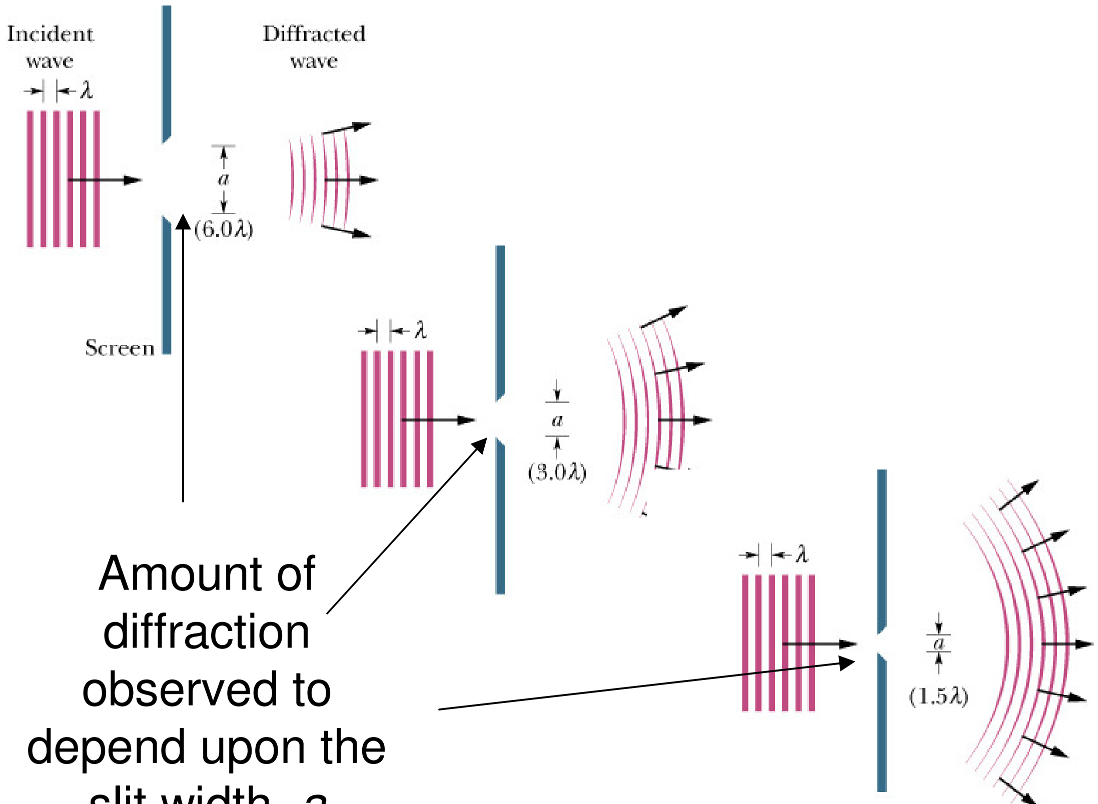
$$\Downarrow$$

$$n = \frac{c}{v} = \frac{c}{f\lambda_n} = \frac{c}{f}\frac{1}{\lambda_n}$$

$$\Rightarrow \boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2}}$$

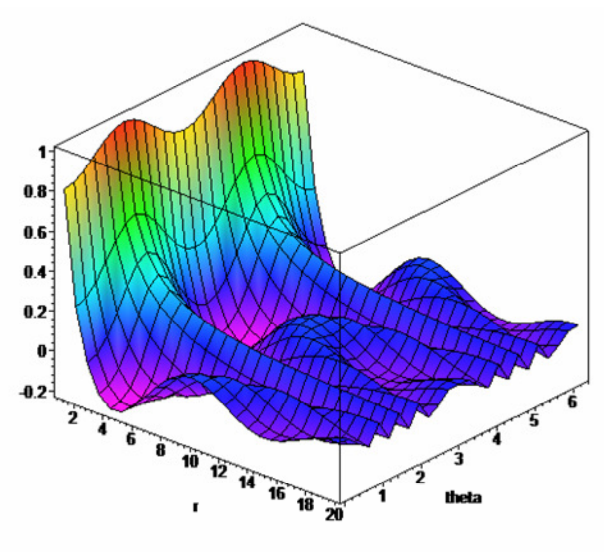
Single-Slit Diffraction

For <u>plane waves</u> entering a single slit, the waves emerging from the slit start spreading out or *diffracting*:



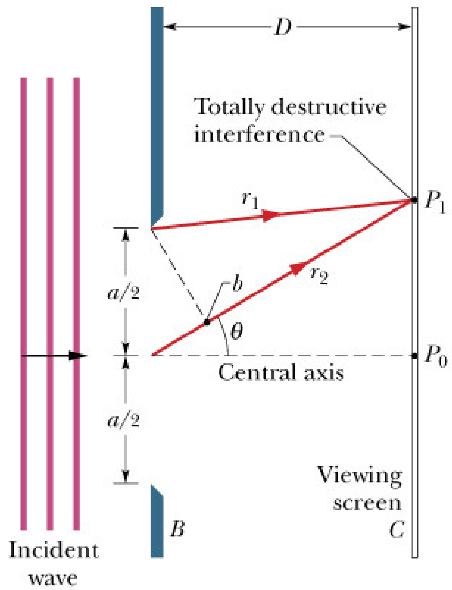
Amount of diffraction observed to depend upon the slit width, *a*

A Plane Wave

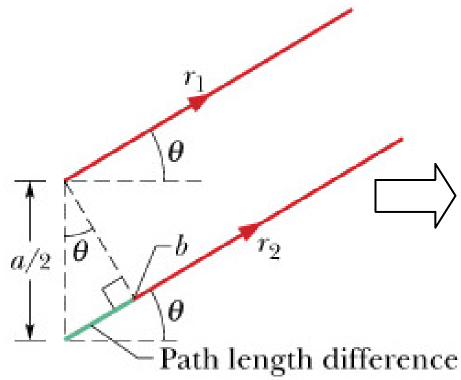


$$\psi(r,\theta) = A\frac{e^{-2\pi i k r\cos\theta}}{r}$$

Locating Single-Slit Diffraction Minima



Setting path length difference to $\lambda/2$ for each pair of rays, we obtain the first dark fringes at:



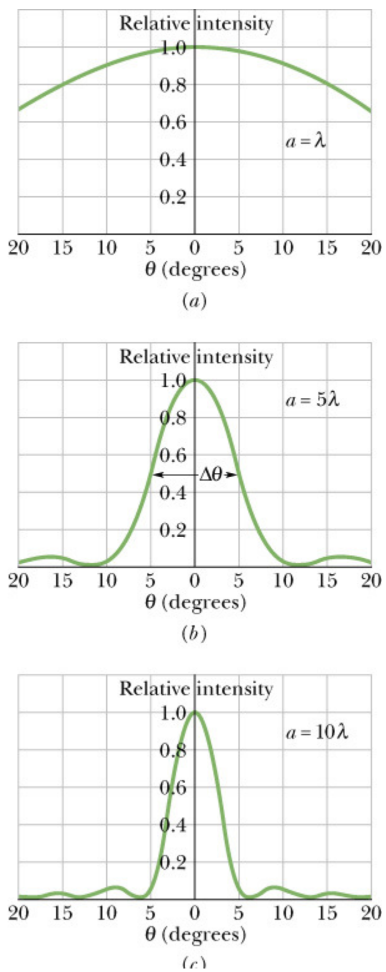
$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}$$

(first minimum) $$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = \lambda$$

Dividing the slit into increasingly larger even numbers of zones, we can find higher order minima:

$$\boxed{a\sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3\ldots}$$

Single-Slit Diffraction Intensity



Width of intensity inversely proportional to slit width, *a*

Pattern produced from a single slit.

The intensity at the screen due to a single slit is:

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$\text{where} \quad \alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta$$

Minima in the intensity occur when:

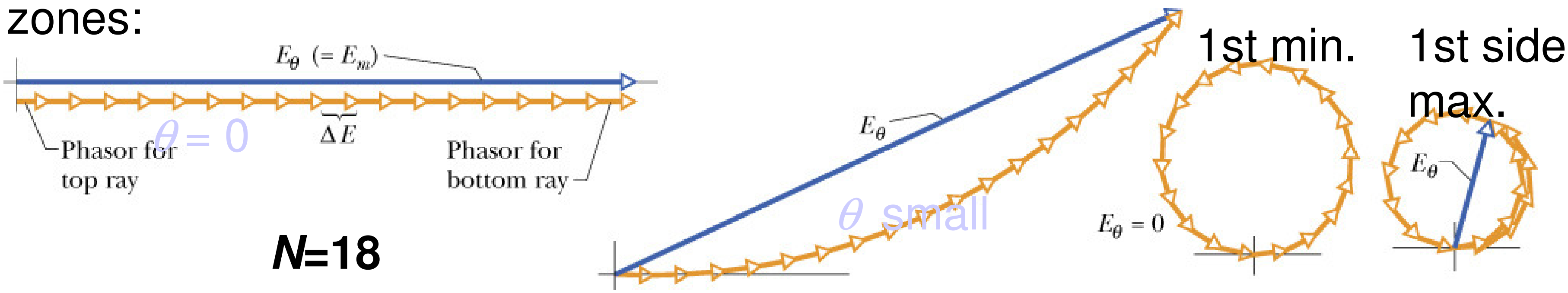
$$a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3 \ldots$$

Intensity in Single-Slit Diffraction, Qualitatively

To obtain the locations of the minima, the slit was equally divided into *N* zones, each with width Δx. Each zone acts as a source of Huygens wavelets. Now these zones can be superimposed at the screen to obtain the intensity as a function of θ, the angle to the central axis.

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix} \Longrightarrow \Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta)$$

To find the net electric field E_θ (intensity $\propto E_\theta^2$) at point *P* on the screen, we need the phase relationships among the wavelets arriving from different zones:



Activity – Diffraction

- **Examine single-slit parameters**
  - Width of central bright fringe
  - Width of secondary bright fringe

Your Name (**Print**): ____________________ Date: __________
Group Members: ____________________
____________________ Group: __________

Diffraction

Caution: Class IV lasers (mostly harmless) are used in this experiment. Under no circumstances should you look directly into the laser, even if it seems like a really good idea at the time and your lab partners bet you a quarter it won't hurt. Laser light reflected from non-metallic surfaces is safe, but be careful with reflective surfaces. Please help save batteries and turn them off if you're doing calculations for any length of time.

The single-slit diffraction geometry is shown in Figure 1. As discussed in class, the first position of fully destructive interference (the center of a dark fringe) occurs when the light from the top half of a slit of width a interferes destructively with the light from the bottom half of the slit. In general, the angular positions of the dark fringes θ_m are determined by

$$a\sin\theta_m = m\lambda, \text{ where } m = \pm1, \pm2, \pm3, \cdots \textbf{ but not } m = 0 \qquad (1)$$

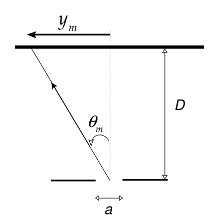
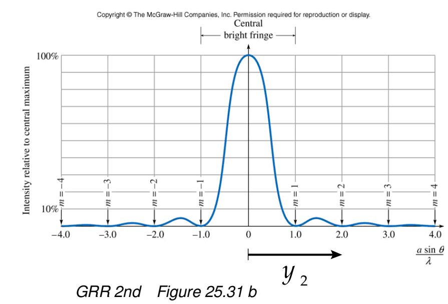


Figure 1 Single-slit Diffraction Geometry



GRR 2nd Figure 25.31 b

Instead of measuring the angle θ_m directly, you will measure the linear distance y_m along the screen from the center of the pattern ($\theta = 0$) to the center of the m^{th} dark fringe. The angle θ_m is related to the distance y_m and the distance D from the slit to the screen; specifically,

$$\tan\theta_m = \frac{y_m}{D}. \qquad (2)$$

If θ_m is small, then

$$\sin\theta_m \approx \tan\theta_m = \frac{y_m}{D}. \qquad (3)$$